8.5

Exponential Growth Functions

What *you should learn*

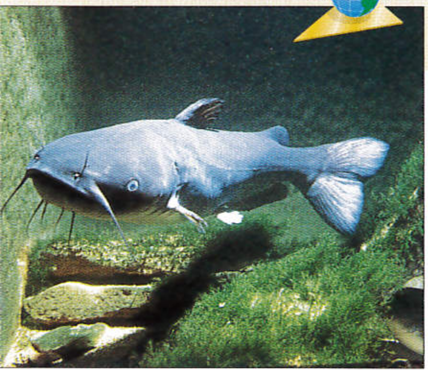
GOAL 1 Write and use models for exponential growth.

GOAL 2 Graph models for exponential growth.

Why *you should learn it*

▼ To solve **real-life** problems such as finding the weight of a channel catfish in **Example 2**.

REAL LIFE



GOAL 1 WRITING EXPONENTIAL GROWTH MODELS

A quantity is *growing exponentially* if it increases by the same percent in each unit of time. This is called **exponential growth**.

EXPONENTIAL GROWTH MODEL

C is the **initial amount.**

t is the **time period.**

$$y = C(1 + r)^t$$

$(1 + r)$ is the **growth factor,** r is the **growth rate.**

The **percent of increase** is $100r$.

EXAMPLE 1 *Finding the Balance in an Account*

COMPOUND INTEREST You deposit \$500 in an account that pays 8% annual interest compounded yearly. What is the account balance after 6 years?

SOLUTION

Method 1 **SOLVE A SIMPLER PROBLEM**

Find the account balance A_1 after 1 year and multiply by the growth factor to find the balance for each of the following years. The growth rate is 0.08, so the growth factor is $1 + 0.08 = 1.08$.

$A_1 = 500(1.08) = 540$ **Balance after 1 year**

$A_2 = 500(1.08)(1.08) = 583.20$ **Balance after 2 years**

$A_3 = 500(1.08)(1.08)(1.08) = 629.856$ **Balance after 3 years**

⋮ ⋮

$A_6 = 500(1.08)^6 \approx 793.437$ **Balance after 6 years**

▶ The account balance after 6 years will be about \$793.44.

Method 2 **USE A FORMULA**

Use the exponential growth model to find the account balance A. The growth rate is 0.08. The initial value is 500.

$A = C(1 + r)^t$ **Exponential growth model**

$= 500(1 + 0.08)^6$ **Substitute 500 for C, 0.08 for r, and 6 for t.**

$= 500(1.08)^6$ **Simplify.**

≈ 793.437 **Evaluate.**

▶ The balance after 6 years will be about \$793.44.

EXAMPLE 2 *Writing an Exponential Growth Model*

REAL LIFE *Catfish Growth*

A newly hatched channel catfish typically weighs about 0.3 gram. During the first six weeks of life, its growth is approximately exponential, increasing by about 10% each day.

a. Write a model for the weight during the first six weeks.

b. Find the weight at the end of six weeks.

SOLUTION

a. Let W represent the weight in grams, and let t represent the time in days. The initial weight is $C = 0.3$ and the growth rate is 0.10.

$$W = C(1 + r)^t \qquad \text{Exponential growth model}$$

$$= 0.3(1 + 0.10)^t \qquad \text{Substitute 0.3 for } C \text{ and 0.10 for } r.$$

$$= 0.3(1.1)^t \qquad \text{Simplify.}$$

b. To find the weight at the end of 6 weeks (or 42 days), substitute 42 for t.

$$W = 0.3(1.1)^{42} \qquad \text{Substitute.}$$

$$\approx 16.42910977 \qquad \text{Use a calculator.}$$

$$\approx 16.4 \qquad \text{Round to the nearest tenth.}$$

▶ The weight is about 16.4 grams.

EXAMPLE 3 *Writing an Exponential Growth Model*

REAL LIFE *Population Growth*

A population of 20 rabbits is released into a wild-life region. The population triples each year for 5 years.

a. What is the percent of increase each year?

b. What is the population after 5 years?

SOLUTION

a. The population triples each year, so the growth factor is 3.

$$1 + r = 3$$

▶ So, the growth rate r is 2 and the percent of increase each year is 200%.

b. After 5 years, the population is

$$P = C(1 + r)^t \qquad \text{Exponential growth model}$$

$$= 20(1 + 2)^5 \qquad \text{Substitute for } C, r, \text{ and } t.$$

$$= 20 \cdot 3^5 \qquad \text{Simplify.}$$

$$= 4860 \qquad \text{Evaluate.}$$

▶ There will be about 4860 rabbits after 5 years.

STUDENT HELP

Study Tip
Notice that the growth factor and the percent of increase are not the same. In Example 3, the growth factor is 3 but the percent of increase is 200%.

GOAL 2 Graphing Exponential Growth Models

You can graph exponential growth models in the same way you graphed exponential functions in Lesson 8.2.

REAL LIFE

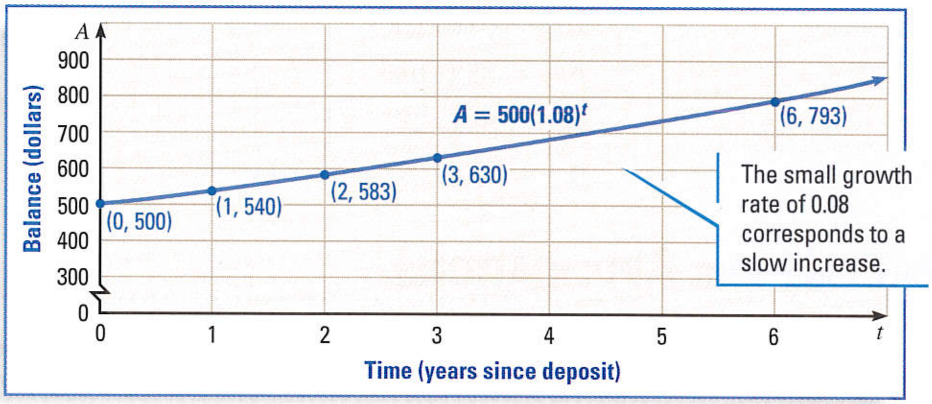
Compound Interest

EXAMPLE 4 *A Model with a Small Growth Factor*

Graph the exponential growth model in Example 1.

Solution

Use the values found in Method 1 of Example 1 to plot points in a coordinate plane. Then, draw a smooth curve through the points.



REAL LIFE

Population Growth

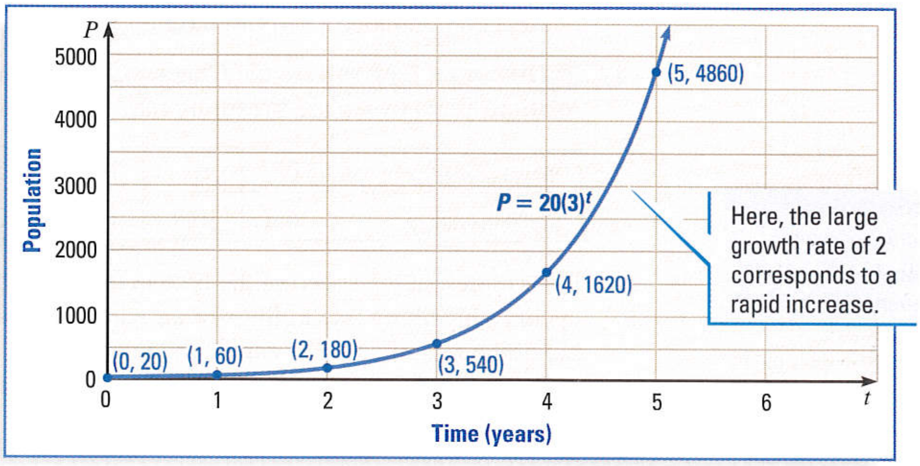
EXAMPLE 5 *A Model with a Large Growth Factor*

Graph the exponential growth model in Example 3.

Solution

Make a table of values, plot the points in a coordinate plane, and draw a smooth curve through the points.

t	0	1	2	3	4	5
P	20	60	180	540	1620	4860



GUIDED PRACTICE

Vocabulary Check ✓

1. In the exponential growth model, $y = C(1 + r)^t$, C is the _?_ and $(1 + r)$ is the _?_.

Concept Check ✓

2. Look back at Example 1. Suppose that you got a 12% annual interest rate. Write a new exponential growth model for the balance in the account.

3. Look back at Example 3. Suppose that the rabbit population doubled every year for 5 years. What was the percent of increase? the growth factor?

Skill Check ✓

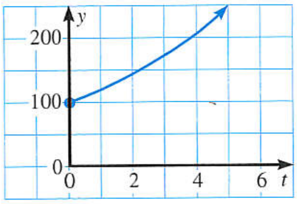
4. **ACCOUNT BALANCE** You deposit \$500 in an account that pays 4% interest compounded yearly. What is the balance after 5 years? after 10 years?

5. **CHOOSE A MODEL** Which model best represents the growth curve shown in the graph at the right?

A. $y = 100(1.08)^t$

B. $y = 100(1.2)^t$

C. $y = 200(1.08)^t$



PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

FIND THE BALANCE **You deposit \$1400 in an account that pays 6% interest compounded yearly. Find the balance for the given time period.**

6. 5 years **7.** 8 years **8.** 12 years **9.** 20 years

FIND THE BALANCE **Find the balance after 5 years of an account that pays 4.8% interest compounded yearly given the following investment amounts.**

10. \$250 **11.** \$300 **12.** \$350 **13.** \$400

14. **ACCOUNT BALANCE** A principal of \$200 is deposited in an account that pays 4.2% interest compounded yearly. Find the balance after 5 years.

15. **INVESTING** How much must you deposit in an account that pays 3.5% interest compounded yearly to have a balance of \$400 after 6 years?

16. **INVESTING** How much must you deposit in an account that pays 6% interest compounded yearly to have a balance of \$1000 after 8 years?

17. **BICYCLE RACING** In the Chapter Opener you learned that there is a relationship between the breathing rate of a cyclist and the bicycle speed.

Bicycle speed, x	0	5	10	15	20
Breathing rate, y	6.4	10.7	18.1	30.5	51.4

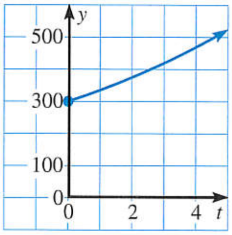
Let x represent the speed of the bike in miles per hour, and let y represent the cyclist's breathing rate in liters of air taken into the lungs per minute. The breathing rate of a cyclist can be modeled by $y = 6.37(1.11)^x$. What is the cyclist's breathing rate if the bike is traveling 19 miles per hour? 25 miles per hour?

STUDENT HELP

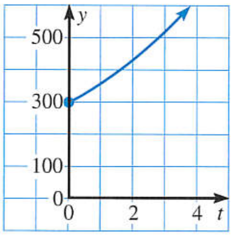
HOMEWORK HELP
Example 1: Exs. 6–17
Example 2: Exs. 21, 22
Example 3: Ex. 24
Example 4: Exs. 18–20
Example 5: Exs. 25–28

MATCHING THE GRAPH **Match the description with its graph.**

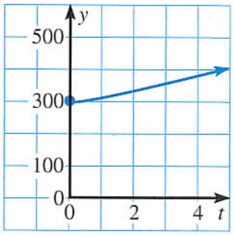
A.



B.



C.



18. Deposit: \$300
Annual rate: 6%

19. Deposit: \$300
Annual rate: 12%

20. Deposit: \$300
Annual rate: 20%

BUSINESS **In Exercises 21 and 22, write an exponential growth model.**

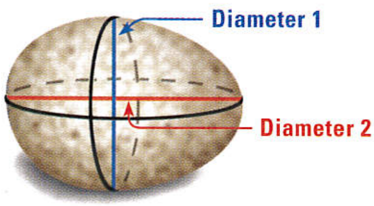
21. A business had a \$10,000 profit in 1990. Then the profit increased by 25% per year for the next 10 years.

22. A business had a \$20,000 profit in 1990. Then the profit increased by 20% per year for the next 10 years.

23. **VISUAL THINKING** Graph the exponential growth models in Exercises 21 and 22 in the same coordinate plane. Which business would you rather own? Explain.

24. **POPULATION GROWTH** A population of 30 mice is released in a wildlife region. The population doubles each year for 4 years. What is the population after 4 years?

BIOLOGY CONNECTION **In Exercises 25–28, use the following information.**
The average of the diameters of the two cross-sections shown can be related to the length of the adult bird. This average for a robin's egg is 25 millimeters (mm); for a bluebird, 19 mm; and for a chickadee, 13 mm. The graph shows the relationship between the average A of the two diameters and the length L (in centimeters) of an adult bird.

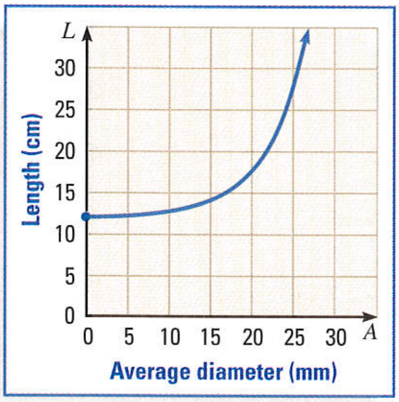


25. **CHOOSING A MODEL** Which model best represents the growth curve shown in the graph?

A. $L = (0.09)(1.23)^A$

B. $L = 12 + (0.09)(1.23)^A$

C. $L = 12 + [(0.09)(1.23)]^A$



26. Estimate the length of an adult bluebird from the graph.

27. Estimate the length of an adult bluebird using the exponential growth model. How does it compare to the length you found in Exercise 26?

28. Use the exponential growth model to find the ratio of the length of a chickadee to the length of a robin.

FOCUS ON APPLICATIONS

REAL LIFE **CHICKADEES** Black-capped Chickadees survive freezing weather by lowering their body temperatures at night, entering a state of controlled hypothermia.

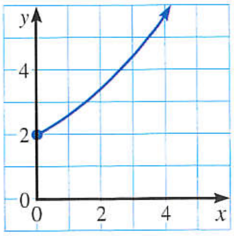
INTERNET **APPLICATION LINK**
www.mcdougallittell.com

Test Preparation

29. **MULTIPLE CHOICE** The hourly rate of your new job is \$5.00 per hour. You expect a raise of 9% each year. At the end of your first year, you receive your first raise. What will your hourly rate be at the end of your fifth year?

(A) \$5.45 (B) \$7.25 (C) \$7.69 (D) \$7.76

30. **MULTIPLE CHOICE** What is the equation of the graph?



(A) $y = 2(0.88)^x$ (B) $y = 4(0.88)^x$

(C) $y = 2(1.3)^x$ (D) $y = 4(1.3)^x$

★ Challenge

31. **EXTENSION: COMPOUND INTEREST** What is the value of an \$8000 investment after 5 years if it earns 8% annual interest compounded quarterly? To solve, use the compound interest formula, $A = P(1 + i)^n$, where P is the original value of the investment, i is the interest rate per compounding period, n is the total number of compounding periods, and A is the value of the investment after n periods.

a. What is the interest rate per quarter?

b. How many compounding periods (quarters) are there in 5 years?

c. Use the formula $A = P(1 + i)^n$ to find the value of the investment after 5 years.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

PERCENT OF A NUMBER **Find the percent of a number.** (Skills Review page 786)

32. 12% of 56
33. 75% of 235
34. 1.25% of 90
35. 200% of 130
36. 2% of 105
37. 0.8% of 120

EVALUATING VARIABLE EXPRESSIONS **Evaluate the expression.** (Review 1.3)

38. $24 + m^3$ when $m = 5$
39. $\frac{a^2 - b^2}{ab}$ when $a = 3$ and $b = 5$
40. $x^6 - 1$ when $x = 1.2$
41. $3y^4 + 15y$ when $y = -0.02$
42. $(1 - x)^t$ when $x = 0.5$ and $t = 3$
43. $\frac{(1 - x)^t}{2}$ when $x = 0.09$ and $t = 2$

44. **BREAKFAST** You are in charge of bringing breakfast for your scout troop. You buy 6 bagels and 8 donuts for a total of \$4.10. Then you decide to buy 3 extra of each for a total of \$1.80. How much did each bagel and donut cost? (Review 7.4)

SOLVING EQUATIONS **Solve the equation.** (Review 3.3, 3.4, 3.6)

45. $-2(7 - 5x) = 10$
46. $25 - (6x + 5) = 4(3x - 5) + 4$
47. $\frac{3}{2}(8m - 30) = -3m$
48. $1.4(6.4y - 3.5) = -9.54y + 22.85$

Investigating Exponential Decay

SET UP
Work in a small group.

MATERIALS
- 100 pennies
- cup

▶ QUESTION

Can an exponential decay model show a decreasing amount?

▶ EXPLORING THE CONCEPT

1. Make a table to record your results.

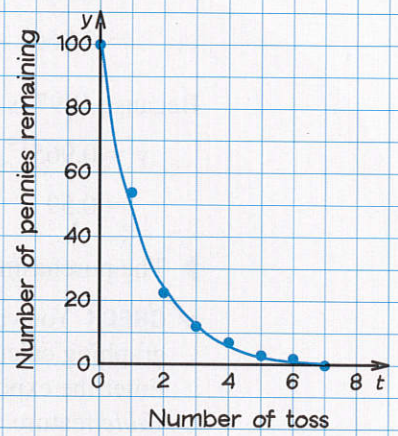
Number of toss	0	1	2	3	4	5	6	7
Number of pennies remaining	?	?	?	?	?	?	?	?

2. Place the pennies in the cup. Shake the pennies in the cup then spill them onto a flat surface. Remove all of the pennies that land face up. Count and record the number of pennies remaining.

3. Repeat **Step 2** until there are no pennies left in the cup.

4. Make a scatter plot of the data you have collected.

Number of toss	0	1	2	3	4	5	6	7
Number of pennies remaining	100	54	23	12	7	3	2	0



▶ DRAWING CONCLUSIONS

1. Describe any patterns suggested by the scatter plot.
2. Look at your data and scatter plot to complete the following statement.

> Each time the cup is emptied, the number of pennies you remove is about __?__ of the number in the cup.

3. Write an exponential equation to model this situation.
4. Compare your equations and graphs with those of other groups. Describe any patterns you see.

8.6 Exponential Decay Functions

What *you should learn*

GOAL 1 Write and use models for exponential decay.

GOAL 2 Graph models for exponential decay.

Why *you should learn it*

▼ To solve **real-life** problems, such as comparing the buying power of a dollar in different years as in **Example 1**.

REAL LIFE

GOAL 1 WRITING EXPONENTIAL DECAY MODELS

In Lesson 8.5, you learned that a quantity is *growing exponentially* if it *increases* by the same percent in each unit of time. A quantity is *decreasing exponentially* if it *decreases* by the same percent in each unit of time. This is called **exponential decay**.

EXPONENTIAL DECAY MODEL

$$y = C(1 - r)^t$$

C is the **initial amount**.

t is the **time period**.

$(1 - r)$ is the **decay factor**, *r* is the **decay rate**. To assure $1 > (1 - r) > 0$, it is necessary that $0 < r < 1$.

The **percent of decrease** is $100r$.

EXAMPLE 1 *Writing an Exponential Decay Model*

PURCHASING POWER From 1982 through 1997, the purchasing power of a dollar decreased by about 3.5% per year. Using 1982 as the base for comparison, what was the purchasing power of a dollar in 1997?

▶ Source: *Statistical Abstract of the United States:* 1998.

SOLUTION Let y represent the purchasing power and let $t = 0$ represent the year 1982. The initial amount is \$1. Use an exponential decay model.

$y = C(1 - r)^t$ — **Exponential decay model**

$= (1)(1 - 0.035)^t$ — **Substitute 1 for *C* and 0.035 for *r*.**

$= 0.965^t$ — **Simplify.**

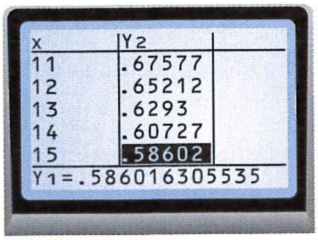
Because 1997 is 15 years after 1982, substitute 15 for t.

$y = 0.965^{15}$ — **Substitute 15 for *t*.**

≈ 0.59 — **Use a calculator.**

▶ The purchasing power of a dollar in 1997 compared to 1982 was \$.59.

✓**CHECK** You can check your result by using a graphing calculator to make a table of values. Enter the exponential decay model and use the *Table* feature. Notice that as the prices *inflate*, the purchasing power of a dollar *deflates*.



REAL LIFE
Depreciation

EXAMPLE 2 *Writing an Exponential Decay Model*

You bought a used car for \$18,000. The value of the car will be less each year because of depreciation. The car depreciates (loses value) at the rate of 12% per year.

a. Write an exponential decay model to represent this situation.

b. Estimate the value of your car in 8 years.

SOLUTION

STUDENT HELP

Skills Review
For help with writing a percent as a decimal, see page 784.

a. The initial value C is \$18,000. The decay rate r is 0.12. Let y be the value and let t be the age of the car in years.

$y = C(1 - r)^t$ — **Exponential decay model**

$= 18{,}000(1 - 0.12)^t$ — **Substitute 18,000 for *C* and 0.12 for *r*.**

$= 18{,}000(0.88)^t$ — **Simplify.**

▶ The exponential decay model is $y = 18{,}000(0.88)^t$.

b. To find the value in 8 years, substitute 8 for t.

$y = 18{,}000(0.88)^t$ — **Exponential decay model**

$= 18{,}000(0.88)^8$ — **Substitute 8 for *t*.**

≈ 6473 — **Use a calculator.**

▶ According to this model, the value of your car in 8 years will be about \$6473.

REAL LIFE
Depreciation

EXAMPLE 3 *Making a List to Verify a Model*

Verify the model you found in Example 2. Find the value of the car for each year by multiplying the value in the previous year by the decay factor.

SOLUTION The decay rate is 0.12. Decay factor = 1 − 0.12 = 0.88.

Year	Value
0	18,000
1	0.88(18,000) = 15,840
2	0.88(15,840) ≈ 13,939
3	0.88(13,939) ≈ 12,266
4	0.88(12,266) ≈ 10,794
5	0.88(10,794) ≈ 9499
6	0.88(9499) ≈ 8359
7	0.88(8359) ≈ 7356
8	0.88(7356) ≈ 6473

← **Initial value of the car**

From the list you can see that the value of the car in 8 years will be about \$6473, which is consistent with your model.

GOAL 2 GRAPHING EXPONENTIAL DECAY MODELS

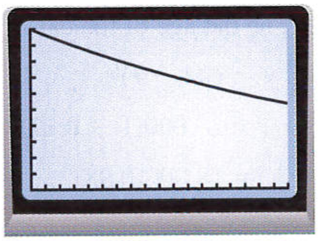
REAL LIFE
Purchasing Power

EXAMPLE 4 *Graphing an Exponential Decay Model*

Graph the exponential decay model in Example 1.

SOLUTION

Use your graphing calculator to graph the model. Set the viewing rectangle so that $0 \le x \le 20$ and $0 \le y \le 1$.



REAL LIFE
Depreciation

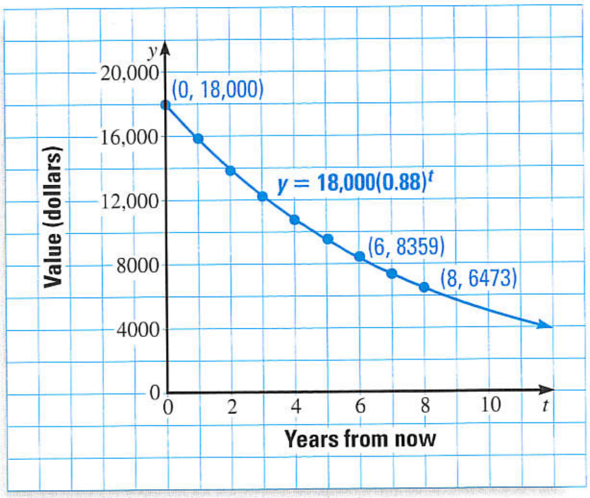
EXAMPLE 5 *Graphing an Exponential Decay Model*

a. Graph the exponential decay model in Example 2.

b. Use the graph to estimate the value of your car in 10 years.

SOLUTION

a. Use the table of values in Example 3. Plot the points in a coordinate plane, and draw a smooth curve through the points.



b. From the graph, the value of your car in 10 years will be about \$5000.

....

An exponential model $y = a \cdot b^t$ represents exponential growth if $b > 1$ and exponential decay if $0 < b < 1$.

EXAMPLE 6 *Comparing Growth and Decay Models*

Classify each model as *exponential growth* or *exponential decay*. In each case identify the growth or decay factor and the percent of increase or percent of decrease per time period. Then graph each model.

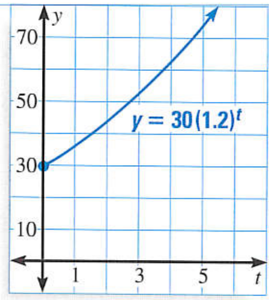
a. $y = 30(1.2)^t$ **b.** $y = 30\left(\frac{3}{5}\right)^t$

SOLUTION

a. Because $1.2 > 1$, the model $y = 30(1.2)^t$ is an exponential growth model.

The growth factor $(1 + r)$ is 1.20.

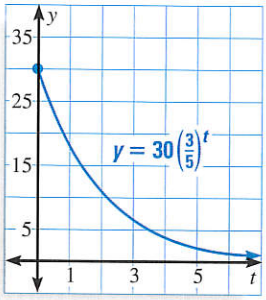
The growth rate is 0.2, so the percent of increase is 20%.



b. Because $0 < \frac{3}{5} < 1$, the model $y = 30\left(\frac{3}{5}\right)^t$ is an exponential decay model.

The decay factor $(1 - r) = \frac{3}{5}$.

The decay rate is $\frac{2}{5}$ or 0.4, so the percent of decrease is 40%.

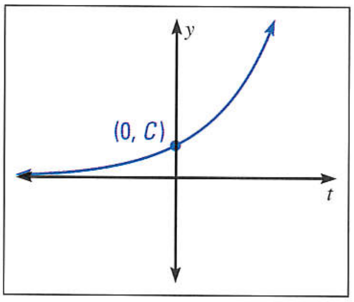


CONCEPT SUMMARY — EXPONENTIAL GROWTH AND DECAY MODELS

EXPONENTIAL GROWTH MODEL

$y = C(1 + r)^t$ ← **time period**

initial amount (C) **growth factor** (1 + r)

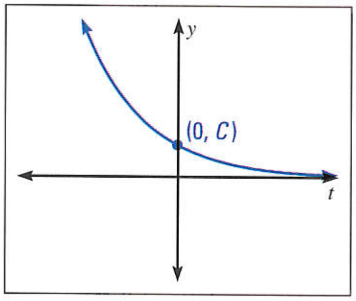


$1 + r > 1$

EXPONENTIAL DECAY MODEL

$y = C(1 - r)^t$ ← **time period**

initial amount (C) **decay factor** (1 − r)



$0 < 1 - r < 1$

GUIDED PRACTICE

Vocabulary Check ✓

1. In the exponential decay model, $y = C(1 - r)^t$, what is the decay factor?

Concept Check ✓

2. Is $y = 1.02^t$ an exponential decay model? Explain.

3. Look back at Example 2. Suppose that the car was depreciating at the rate of 20% per year. Write a new exponential decay model.

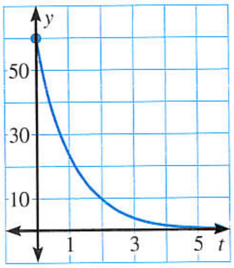
Skill Check ✓

CAR VALUE **You buy a used car for \$7000. The car depreciates at the rate of 6% per year. Find the value of the car in the given years.**

4. 2 years
5. 5 years
6. 8 years
7. 10 years

8. **BUSINESS DECLINE** A business earned \$85,000 in 1990. Then its earnings decreased by 2% each year for 10 years. Write an exponential decay model for the earnings E in year t. Let $t = 0$ represent 1990.

9. **CHOOSE A MODEL** Which model best represents the decay curve shown in the graph at the right?



A. $y = 60(0.80)^t$

B. $y = 60(1.20)^t$

C. $y = 60(0.40)^t$

PRACTICE AND APPLICATIONS

STUDENT HELP

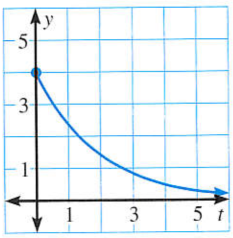
Extra Practice to help you master skills is on p. 804.

TRUCK VALUE **You buy a used truck for \$20,000. It depreciates at the rate of 15% per year. Find the value of the truck in the given years.**

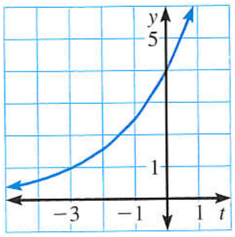
10. 3 years
11. 8 years
12. 10 years
13. 12 years

MATCHING THE GRAPH **Match the equation with its graph.**

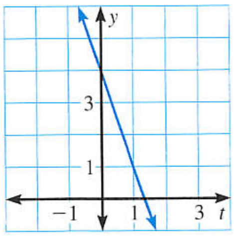
A.



B.



C.



14. $y = 4 - 3t$
15. $y = 4(1.6)^t$
16. $y = 4(0.6)^t$

STUDENT HELP

HOMEWORK HELP
Examples 1, 2: Exs. 10–13, 23–29
Example 3: Exs. 30–33
Examples 4, 5: Exs. 14–16, 30, 33
Example 6: Exs. 14–22

RECOGNIZING MODELS **Classify the model as *exponential growth* or *exponential decay*. Identify the growth or decay factor and the percent of increase or decrease per time period.**

17. $y = 24(1.18)^t$
18. $y = 14(0.98)^t$
19. $y = 35\left(\frac{5}{4}\right)^t$
20. $y = 112(0.4)^t$
21. $y = 9\left(\frac{2}{5}\right)^t$
22. $y = 97(1.01)^t$

FOCUS ON CAREERS

PHARMACISTS Pharmacists must understand the use, composition, and effects of drugs.

CAREER LINK
www.mcdougallittell.com

SCIENCE CONNECTION **In Exercises 23–25, use the following information.** The concentration of aspirin in a person's bloodstream can be modeled by the equation $y = A(0.8)^t$, where y represents the concentration of aspirin in a person's bloodstream in milligrams (mg), A represents the amount of aspirin taken, and t represents the number of hours since the medication was taken. Find the amount of aspirin remaining in a person's bloodstream at the given dosage.

23. Dosage: 250 mg
Time: after 2 hours

24. Dosage: 500 mg
Time: after 3.5 hours

25. Dosage: 750 mg
Time: after 5 hours

26. **BUYING A TRUCK** You buy a used truck in 1996 for \$10,500. Each year the truck depreciates by 10%. Write an exponential decay model to represent this situation. Then estimate the value of the truck in 10 years.

BASKETBALL **In Exercises 27–29, use the following information.** Each year in the month of March, the NCAA basketball tournament is held to determine the national champion. At the start of the tournament there are 64 teams, and after each round, one half of the remaining teams are eliminated.

27. Write an exponential decay model showing the number of teams N left in the tournament after round t.

28. How many teams remain after 3 rounds? after 4 rounds?

29. **CRITICAL THINKING** If a team won 6 games in a row in the tournament, does it mean that it won the national championship? Explain your reasoning.

30. **SUMMER CAMP** A summer youth camp had a declining enrollment from 1995 to 2000. The enrollment in 1995 was 320 people. Each year for the next five years, the enrollment decreased by 2%. Copy and complete the table showing the enrollment for each year. Sketch a graph of the results.

Year	1995	1996	1997	1998	1999	2000
Enrollment	?	?	?	?	?	?

CABLE CARS **In Exercises 31–33, use the following information. From 1894 to 1903 the number of miles of cable car track decreased by about 10% per year. There were 302 miles of track in 1894.**

31. Write an exponential decay model showing the number of miles M of cable car track left in year t.

32. Copy and complete the table. You may want to use a calculator.

Year	1894	1896	1898	1899	1900	1901	1903
Miles of track	?	?	?	?	?	?	?

33. Sketch a graph of the results.

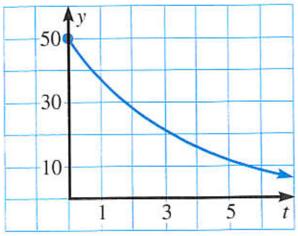
34. **CRITICAL THINKING** A store is having a sale on sweaters. On the first day the price of a sweater is reduced by 20%. The price will be reduced another 20% each day until the sweater is sold. Denise thinks that on the fifth day of the sale the sweater will be free. Is she right? Explain.

Test Preparation

35. **MULTIPLE CHOICE** In 1995, you purchase a parcel of land for \$8000. The value of the land depreciates by 4% every year. What will the approximate value of the land be in 2002?

(A) \$5760 (B) \$5771 (C) \$6012 (D) \$6262

36. **MULTIPLE CHOICE** Which model best represents the decay curve shown in the graph at the right?



(A) $y = 20(1.16)^t$

(B) $y = 50(0.75)^t$

(C) $y = 20(0.75)^t$

(D) $y = 50 + 20(1.16)^t$

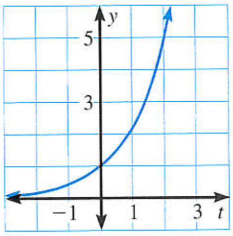
★ Challenge

UNDERSTANDING GRAPHS In Exercises 37–39, use a graphing calculator.

37. Make an input-output table for the equations $y = 4^t$ and $y = \left(\frac{1}{4}\right)^t$. Use -3, -2, -1, 0, 1, 2, and 3 as the input. Then sketch the graph of each equation.

38. **VISUAL THINKING** Interpret the graphs. How are they related?

39. **VISUAL THINKING** The graph at the right is $y = 2.26^t$. Based on the relationships between the graphs in Exercise 37, predict the graph of $y = \left(\frac{1}{2.26}\right)^t$.



EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

VARIABLE EXPRESSIONS Evaluate the expression. (Review 1.3 for 9.1)

40. $4a^2 + 11$ when $a = 5$

41. $c^3 + 6cd$ when $c = 2$ and $d = 1$

42. $b^2 - 4ac$ when $a = 1$, $b = 3$, and $c = 5$

43. $\frac{a^2 - b^2}{2c^2} + 9$ when $a = -3$, $b = 5$, and $c = -2$

SOLVING EQUATIONS Solve the equation. Round the result to the nearest tenth if necessary. (Review 3.6)

44. $12m - 9 = 5m - 2$

45. $5(2x + 2.3) - 11.2 = 6x - 5$

46. $-1.3y + 3.7 = 4.2 - 5.4y$

47. $2.5(3.5p + 6.4) = 18.2p - 6.5$

48. **TELEVISION** A TV station's local news program has 50,000 viewers. The managers of the station plan to increase the number of viewers by 2% per month. Write an exponential growth model to represent the number of viewers in t months. (Review 8.5)

SCIENTIFIC NOTATION **Rewrite in scientific notation.** (Lesson 8.4)

1. 0.011205 **2.** 140,000,000 **3.** 0.000000067 **4.** 30,720,000,000

DECIMAL FORM **Rewrite in decimal form.** (Lesson 8.4)

5. 4.82×10^3 **6.** 5×10^9 **7.** 7.04×10^{-6} **8.** 1.112×10^{-2}

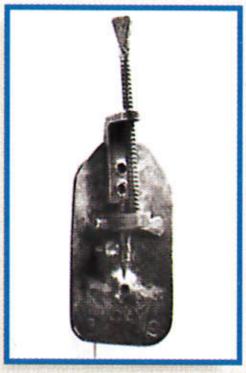
9. **INVESTMENT** In 1995, you bought a baseball card for $50 that you expect to increase in value 5% each year for the next 10 years. Write an exponential growth model and estimate the value of the baseball card in 2002. (Lesson 8.5)

10. **BUYING A CAMPER** You buy a used camper in 1995 for $20,000. Each year the camper depreciates by 15%. Write an exponential decay model to represent this situation. Then estimate the value of the camper in 5 years. (Lesson 8.6)

MATH & History — History of Microscopes

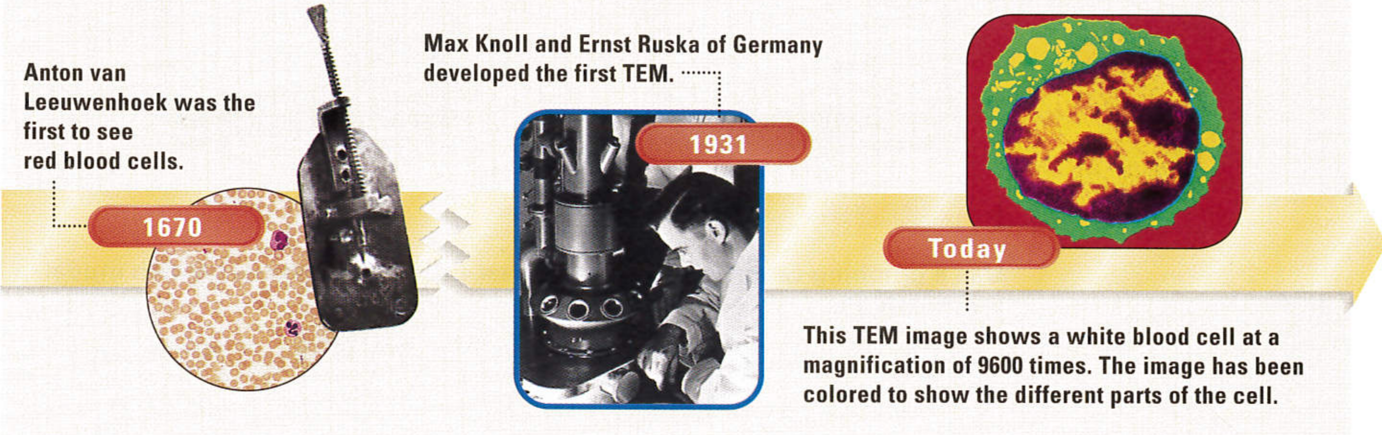
APPLICATION LINK www.mcdougallittell.com

THEN **THE EARLIEST MICROSCOPES** consisted of a single, strongly curved lens mounted on a metal plate. These simple microscopes used visible light to illuminate the object being viewed and could magnify objects as much as 400 times.



NOW **TODAY,** scientists use microscopes that use electrons instead of light. The *transmission electron microscope* (or TEM) can resolve two objects as close together as 0.0000005 millimeter (mm) with magnifications up to 1,000,000 times. At this magnification, a sneaker would be long enough to reach from Boston to New York City.

1. Write 0.0000005 mm in scientific notation.
2. The distance from Boston to New York City is 1,003,200 feet. Write this number in scientific notation.
3. How many times stronger is the magnification of the TEM compared to the earliest microscopes? Write your answer in scientific notation.



Anton van Leeuwenhoek was the first to see red blood cells.

Max Knoll and Ernst Ruska of Germany developed the first TEM.

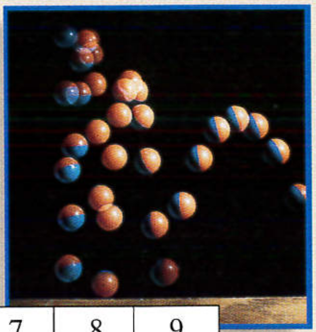
This TEM image shows a white blood cell at a magnification of 9600 times. The image has been colored to show the different parts of the cell.

Fitting Exponential Models

In Chapter 5, you learned that you can use a graphing calculator to find a best-fitting line. A graphing calculator can also be used to find a best-fitting exponential growth or decay model.

EXAMPLE

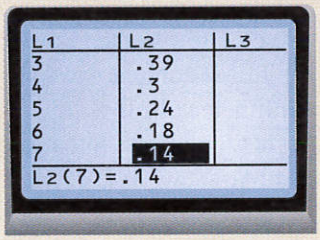
A rubber ball is dropped from a height of 0.82 meter. Using a CBL unit, the height of the ball on each successive bounce was recorded. The x-values represent the bounce and the y-values represent the height. Use a graphing calculator to find an exponential model for these data.



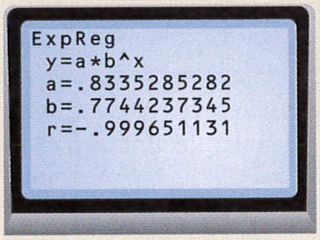
x	0	1	2	3	4	5	6	7	8	9
y	0.82	0.64	0.50	0.39	0.30	0.24	0.18	0.14	0.11	0.08

SOLUTION

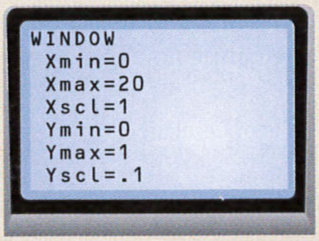
1 Enter the ordered pairs into the graphing calculator. Select L_1 as the x list and L_2 as the y list.



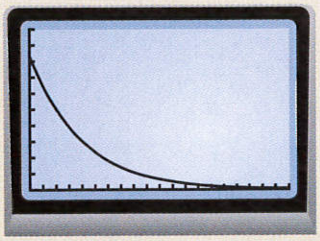
2 Use exponential regression to find an exponential model. The equation $y = 0.8335(0.7744)^x$ is the best-fitting exponential model.



3 Set the viewing rectangle so that $0 \le x \le 20$ and $0 \le y \le 1$.



4 Graph the equation $y = 0.8335(0.7744)^x$.



STUDENT HELP

INTERNET KEYSTROKE HELP

See keystrokes for several models of calculators at www.mcdougallittell.com

EXERCISES

Use a graphing calculator to find the best-fitting exponential growth model for the points.

1. (0, 1), (1, 1.4), (2, 3), (3, 5), (4, 8), (5, 12), (6, 20), (7, 30), (8, 50), (9, 80)

2. (0, 0.5), (2, 0.8), (3, 1), (4, 1.4), (5, 1.8), (6, 2.7), (7, 3.6), (8, 4.9), (9, 7)

CHAPTER 8

Chapter Summary

WHAT did you learn?	*WHY did you learn it?*
Evaluate exponential expressions	
• using multiplication properties of exponents. **(8.1)**	Find the power generated by a windmill. **(p. 454)**
• that have negative and zero exponents. **(8.2)**	Predict a basketball player's average score per game. **(p. 458)**
• using division properties of exponents. **(8.3)**	Estimate the speed of an Olympic rowing team. **(p. 468)**
Convert numbers from scientific notation to decimal form. **(8.4)**	Find the amount of water discharged by the Amazon River each year. **(p. 472)**
Convert numbers from decimal form to scientific notation. **(8.4)**	Find the price per acre of the Alaska purchase. **(p. 472)**
Perform operations with numbers in scientific notation. **(8.4)**	Find how long it takes light to travel from the Sun to Pluto. **(p. 473)**
Use scientific notation in problem solving. **(8.4)**	Estimate the number of heartbeats in a person's life. **(p. 474)**
Use exponential growth models. **(8.5)**	Find the weight of a channel catfish. **(p. 478)**
Use exponential decay models. **(8.6)**	Find the buying power of a dollar. **(p. 484)**
Sketch the graphs of exponential growth and decay models. **(8.5 and 8.6)**	Find the balance on a savings account. **(p. 480)**

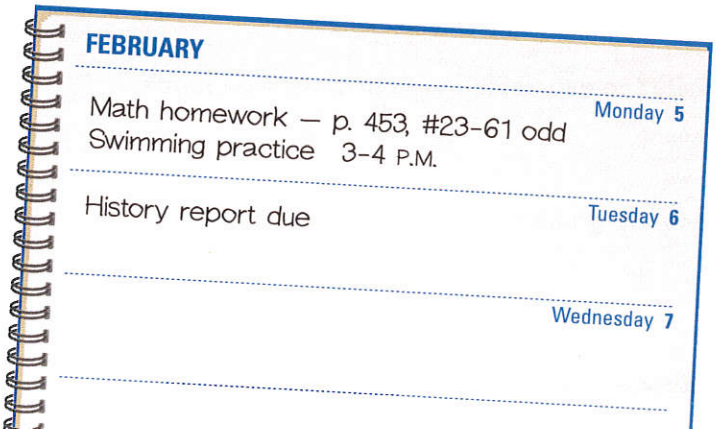
How does Chapter 8 fit into the BIGGER PICTURE of algebra?

In Chapters 3–7, you learned how to solve linear equations such as $4(x - 3y) = 24$. In this chapter you learned how to use the properties of exponents and scientific notation to solve exponential functions such as $y = 42(1.2)^x$. You will need to know how to use the properties of exponents when you solve quadratic equations in Chapter 9 and polynomial equations in Chapter 10.

STUDY STRATEGY

How did you use your schedule?

A schedule that you made using the **Study Strategy** on p. 448 may resemble this one.



CHAPTER 8 Chapter Review

VOCABULARY

- exponential function, p. 458
- scientific notation, p. 470
- exponential growth, p. 477
- growth factor, p. 477
- initial amount, pp. 477, 484
- time period, pp. 474, 484
- percent of increase, p. 477
- growth rate, p. 477
- exponential decay, p. 484
- decay factor, p. 484
- decay rate, p. 484
- percent of decrease, p. 484

8.1 MULTIPLICATION PROPERTIES OF EXPONENTS

Examples on pp. 450–452

EXAMPLES Use the multiplication properties of exponents.

a. $4^2 \cdot 4^7 = 4^{2+7} = 4^9$ — Product of powers property

b. $(5^2)^3 = 5^{2 \cdot 3} = 5^6$ — Power of a power property

c. $(6 \cdot x)^3 = 6^3 \cdot x^3$ — Power of a product property

Simplify the expression.

1. $2^2 \cdot 2^7$ **2.** $(4^3)^2$ **3.** $(3a)^3 \cdot (2a)^2$ **4.** $(w^3x^4y)^2 \cdot (wx^2y^3)^4$

Simplify. Then evaluate the expression when $s = 2$ and $t = 3$.

5. $s^3 \cdot s^4$ **6.** $s^4 \cdot (-t)^3$ **7.** $(s^3 \cdot t)^2$ **8.** $-(st^2)^2$

8.2 ZERO AND NEGATIVE EXPONENTS

Examples on pp. 456–458

EXAMPLES Use zero and negative exponents.

a. $6^0 = 1$ — A nonzero number to the zero power is 1.

b. $7(x^{-3}) = 7\left(\frac{1}{x^3}\right) = \frac{7}{x^3}$ — a^{-n} is the reciprocal of a^n.

Evaluate the expression. Write fractions in simplest form.

9. 5^{-3} **10.** $7^{-4} \cdot 7^6$ **11.** $16\left(\frac{1}{2}\right)^{-1}$ **12.** $2^0 \cdot \left(\frac{1}{4^{-2}}\right)$

Rewrite the expression with positive exponents.

13. x^6y^{-6} **14.** $\frac{1}{5p^8q^{-3}}$ **15.** $(a^2b)^0$ **16.** $(-2y)^{-4}$

Sketch the graph of the exponential function.

17. $y = 4^x$ **18.** $y = \left(\frac{1}{2}\right)^x$ **19.** $y = 3^{-x}$ **20.** $y = \left(\frac{2}{3}\right)^{-x}$

8.3 DIVISION PROPERTIES OF EXPONENTS

Examples on pp. 463–465

EXAMPLES Using the division properties of exponents.

a. $\frac{6^4}{6^2} = 6^{4-2} = 6^2 = 36$ **Quotient of powers property**

b. $\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{8}{27}$ **Power of a quotient property**

Evaluate the expression. Write fractions in simplest form.

21. $\frac{3^2}{3^5}$ **22.** $\frac{5^2}{5^{-2}}$ **23.** $\left(-\frac{4}{9}\right)^2$ **24.** $\left(\frac{10}{7}\right)^{-1}$

Simplify the expression. The simplified expression should have no negative exponents.

25. $\left(\frac{9}{b}\right)^6$ **26.** $\frac{x^{12}}{x^6}$ **27.** $\left(\frac{m^7}{m^4}\right)^2$ **28.** $\frac{(p^2)^3}{(p^2)^5}$

29. $\left(\frac{-9a^2b^2}{3ab}\right)^3$ **30.** $\left(\frac{25a^4b^5}{-5a^2b}\right)^3$ **31.** $\frac{32a^4b^{-2}}{2a^3b^3} \cdot \frac{3a^2b^7}{-2a}$ **32.** $\frac{9x^{-3}y^6}{x^4y^{-5}} \cdot \frac{(3x^2y)^{-2}}{xy^3}$

33. **SALES** From 1994 through 1999, the sales for a national book store increased by about the same percent each year. The sales s (in millions of dollars) for year t can be modeled by $s = 1686(1.17)^t$ where $t = 0$ represents 1994. Find the ratio of 1994 sales to 1999 sales.

8.4 SCIENTIFIC NOTATION

Examples on pp. 470–472

EXAMPLES Rewriting numbers in decimal form and scientific notation.

a. $1.247 \times 10^2 = 124.7$ **Move decimal point right 2 places.**

b. $1.045 \times 10^{-3} = 0.001045$ **Move decimal point left 3 places.**

c. $79{,}500 = 7.95 \times 10^4$ **Move decimal point left 4 places.**

d. $0.0588 = 5.88 \times 10^{-2}$ **Move decimal point right 2 places.**

Rewrite the number in decimal form.

34. 6.667×10^{-3} **35.** 7.68×10^5 **36.** 3.75×10^{-1} **37.** 2×10^{-4}

Rewrite the number in scientific notation.

38. 523,000,000 **39.** 0.000679 **40.** 0.0000000233

41. **SPACE TRAVEL** Astronaut Shannon W. Lucid holds the United States single-mission, space-flight endurance record. Upon completion of her 1996 mission aboard the Russian Space Station *Mir*, Dr. Lucid had traveled 75,200,000 miles. Write 75,200,000 miles in scientific notation.

42. **ASTRONOMY** The distance from Earth to the star Alpha Centauri is about 4.07×10^{13} kilometers (km). Light travels at a speed of about 3.0×10^5 km per second. How long does it take light to travel from this star to Earth?

8.5 EXPONENTIAL GROWTH FUNCTIONS

Examples on pp. 477–479

EXAMPLE

You deposited $1200 in a savings account that pays 9% annual interest compounded yearly. What is the balance after 8 years?

$y = C(1 + r)^t$	**Exponential growth model**
$= 1200(1 + 0.09)^t$	**Substitute 1200 for *C* and 0.09 for *r*.**
$= 1200(1.09)^t$	**Simplify.**

After 8 years, the balance would be

$y = 1200(1.09)^8$	**Substitute 8 for *t*.**
≈ 2391.08	**Use a calculator.**

After 8 years, the balance would be $2391.08.

FITNESS PROGRAM In Exercises 43 and 44, use the following information. You start a walking program. The first week you walk 2 miles. Over the next 9 weeks, you increase your distance 5% per week.

43. Write an exponential growth model to represent the number of miles w you are walking after x weeks.

44. How far are you walking in the tenth week?

8.6 EXPONENTIAL DECAY FUNCTIONS

Examples on pp. 484–487

EXAMPLE

In 1995 you bought a 32-inch television for $600. The television is depreciating at the rate of 8% per year. Write an exponential decay model and estimate the value of the television in 6 years.

$y = C(1 - r)^t$	**Exponential decay model**
$= 600(1 - 0.08)^t$	**Substitute 600 for *C* and 0.08 for *r*.**
$= 600(0.92)^t$	**Simplify.**

After 6 years, the balance would be

$y = 600(0.92)^6$	**Substitute 6 for *t*.**
≈ 363.81	**Use a calculator.**

After 6 years, the television would be worth $363.81.

TENNIS CLUB In Exercises 45 and 46, use the following information. A tennis club had a declining enrollment from 1993 to 2000. The enrollment in 1993 was 125 people. Each year for 7 years, the enrollment decreased by 3%.

45. Write an exponential decay model to represent enrollment e after x weeks.

46. Estimate the enrollment in 2000.

CHAPTER 8

Chapter Test

In Exercises 1–12, simplify the expression. The simplified expression should have no negative exponents.

1. $x^3 \cdot x^4$ **2.** $a^0 \cdot a^4$ **3.** $b^2 \cdot b^{-5}$ **4.** $5y^{-4}$

5. $(x^3)^7$ **6.** $(a^{-2})^3$ **7.** $\frac{n^3}{n^5}$ **8.** $(2b)^3(b^{-4})$

9. $(mn)^2 \cdot n^4$ **10.** $3a^5 \cdot 5a^{-2} \cdot a^3$ **11.** $\left(\frac{x^3}{xy^4}\right)\left(\frac{y}{x}\right)^5$ **12.** $\frac{a^{-1}b^2}{ab} \cdot \frac{a^2b^3}{b^{-2}}$

In Exercises 13–20, evaluate the expression.

13. $5^4 \cdot 5^{-1}$ **14.** 4^{-3} **15.** $(425^2)^0$ **16.** $\left(\frac{5}{2}\right)^{-2}$

17. $\frac{3 \cdot 3^5}{3^4}$ **18.** $\left(\frac{3}{4}\right)^3 \cdot 4^2 \cdot 3^0$ **19.** $(5 \cdot 4)^3 \cdot 5^{-2}$ **20.** $[(-2)^5]^2$

In Exercises 21–24, write the number in decimal form.

21. $4.27 \cdot 10^5$ **22.** $6.283 \cdot 10^{-9}$ **23.** 4.56×10^{10} **24.** 5×10^{-12}

In Exercises 25–28, write the number in scientific notation.

25. 9,875,000 **26.** 0.00125 **27.** 6,557,000,000 **28.** 0.0000000317

In Exercises 29–31, sketch a graph of the equation.

29. $y = 2^x$ **30.** $y = \left(\frac{1}{3}\right)^x$ **31.** $y = 10(1.4)^x$

32. GEOMETRY CONNECTION The volume of a cube is given by $V = s^3$, where s is the length of a side. The cube has a side of length $3a$. What is the volume of the cube if $a = 2$?

33. **SAVINGS ACCOUNT** You started a savings account in 1996. The balance A is given by $A = 400(1.1)^t$, where $t = 0$ represents the year 1996. What is the balance in 2000? in 2003?

34. **SALES** In 1996, you started your own business. In the first year, your sales totaled \$88,500. Then each year for the next 4 years, your sales increased by 20%. Write an exponential growth model to represent this situation. Then estimate your sales in 2001.

35. **RADIOISOTOPES** The amount of time it takes for a radioactive substance to reduce to half of its original amount is called its half-life. The half-life of carbon 11 (^{11}C) is 20 minutes. If you start with 16 grams of ^{11}C, the number of grams remaining after h half-life periods would be $W = 16(0.5)^h$. Copy and complete the table and use the results to sketch the graph.

Half-life periods, h	0	1	2	3	4
Grams remaining, W	?	?	?	?	?

CHAPTER 8 Chapter Standardized Test

TEST-TAKING STRATEGY Be aware of how much time you have left, but keep focused on your work.

1. **MULTIPLE CHOICE** Simplify $7^4 \cdot 7^7$. Write the answer as a power.

Ⓐ 7^3 Ⓑ 7^{11} Ⓒ 7^{28}
Ⓓ 49^{11} Ⓔ 49^{28}

2. **MULTIPLE CHOICE** Evaluate $[(a + 1)^2]^2 \cdot a^3$ when $a = 2$.

Ⓐ 72 Ⓑ 128 Ⓒ 136
Ⓓ 200 Ⓔ 648

3. **MULTIPLE CHOICE** Evaluate $(5^{-3})^2$.

Ⓐ -3125 Ⓑ $-\frac{1}{15,625}$
Ⓒ $\frac{1}{3125}$ Ⓓ $\frac{1}{15,625}$
Ⓔ 15,625

4. **MULTIPLE CHOICE** Evaluate $-8^0 \cdot 2^x \cdot 10^y$ when $x = -2$ and $y = -3$.

Ⓐ $-\frac{1}{4000}$ Ⓑ $-\frac{1}{500}$
Ⓒ $\frac{1}{500}$ Ⓓ $\frac{1}{4000}$
Ⓔ 4000

5. **MULTIPLE CHOICE** Simplify $\frac{4x^2y^2}{4xy} \cdot \frac{8xy^3}{4y}$.

Ⓐ $2x^2y^3$ Ⓑ $4x^2y$
Ⓒ $2xy^2$ Ⓓ $2x^2y^4$
Ⓔ $2xy^3$

6. **MULTIPLE CHOICE** Which expression simplifies to x^3?

Ⓐ $\frac{x^2}{x^5}$ Ⓑ $\frac{x^5}{x^{-2}}$
Ⓒ $\frac{x^5}{x^2}$ Ⓓ $x^5 \cdot x^2$
Ⓔ $x^5 - x^2$

7. **MULTIPLE CHOICE** Which of the following numbers *is not* written in scientific notation?

Ⓐ 8.62×10^4 Ⓑ 2.12×10^{-12}
Ⓒ 21.2×10^{-5} Ⓓ 9.9132×10^{-1}
Ⓔ 2.0001×10^{-3}

8. **MULTIPLE CHOICE** Rewrite 3.6×10^{-6} in decimal form.

Ⓐ 0.000036 Ⓑ 3,600,000
Ⓒ 0.00000036 Ⓓ 36,000,000
Ⓔ 0.0000036

9. **MULTIPLE CHOICE** Evaluate the product $(6.2 \times 10^4) \cdot (2.4 \times 10^5)$. Write the result in scientific notation.

Ⓐ 1.488×10^8 Ⓑ 1.488×10^{10}
Ⓒ 14.88×10^1 Ⓓ 14.88×10^{10}
Ⓔ 14.88×10^{20}

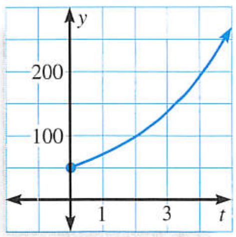
10. **MULTIPLE CHOICE** Evaluate $\frac{(2.3622 \times 10^4)}{(3.81 \times 10^{-3})}$.

Write the result in scientific notation.

Ⓐ 6.2×10^6 Ⓑ 6.2×10^8
Ⓒ 0.62×10^6 Ⓓ 6.2×10^1
Ⓔ 6.2×10^{-12}

11. **MULTIPLE CHOICE** Which model best represents the growth curve shown below?

Ⓐ $y = 50(1.4)^t$
Ⓑ $y = 100(1.4)^t$
Ⓒ $y = 50(1.12)^t$
Ⓓ $y = 50(1.4)^{-t}$
Ⓔ $y = 200(1.08)^t$



12. **MULTIPLE CHOICE** You deposit \$450 into a savings account that pays 6% interest compounded yearly. How much money is in the account after 6 years? Assume you make no more deposits or withdrawals.

Ⓐ \$477.00 Ⓑ \$602.20
Ⓒ \$638.33 Ⓓ \$676.63
Ⓔ \$693.33

13. **MULTIPLE CHOICE** In 1994 you bought a rare stamp for \$500 that you expect to increase in value 12% each year for the next 15 years. Write an exponential growth model and estimate the value of the stamp in 2002.

(A) \$881.17 (B) \$986.91 (C) \$1557.92 (D) \$1237.98 (E) \$2000

QUANTITATIVE COMPARISON In Exercises 14–16, evaluate each function. Then choose the statement below that is true about the given values of *y*.

A. The value of y in column A is greater.

B. The value of y in column B is greater.

C. The two values of y are equal.

D. The relationship cannot be determined from the given information.

		Column A	Column B
14.	When $x = 3$,	$y = 3x$	$y = 3^x$
15.	When $x = -2$,	$y = 3^x$	$y = \left(\frac{1}{3}\right)^x$
16.	When $x = 1$,	$y = 3^{-x}$	$y = \left(\frac{1}{3}\right)^x$

17. **MULTIPLE CHOICE** The concentration of an allergy medication in a person's bloodstream in nanograms per milliliter (ng/mL) can be modeled by the equation $y = 263(0.92)^t$, where t represents the number of hours since the medication was taken. What is the concentration of the medication remaining in the person's bloodstream after 4 hours?

(A) 263 ng/mL (B) 222 ng/mL (C) 205 ng/mL (D) 188 ng/mL (E) 170 ng/mL

18. **MULTIPLE CHOICE** A business had a profit of \$142,000 in 1994. Then its profit decreased by 8% each year for the next 6 years. Which exponential decay model would you use to find how much the business earned in the year 2000? Let E represent the earnings and let t represent the year.

(A) $E = 142{,}000(0.08)^t$ (B) $E = 142{,}000(0.96)^t$ (C) $E = 142{,}000(0.92)^t$

(D) $t = 142{,}000(0.08)^E$ (E) $E = 0.92(142{,}000)^t$

19. **MULTIPLE CHOICE** Which models below are exponential decay models?

I $y = 1.25^t$ **II** $y = 0.97^t$ **III** $y = \left(\frac{4}{3}\right)^t$ **IV** $y = \left(\frac{2}{3}\right)^t$

(A) I and II (B) III and IV (C) II and III (D) I and III (E) II and IV

20. **MULTI-STEP PROBLEM** The population in one midwestern town was tracked over several years. Based on the data for the town, population experts determined that the population, in thousands of people, could be represented by the expression $2(1.175)^t$, where t is the number of years from now.

a. What is the population this year?

b. What is the estimated population 5 years from now?

c. What was the population 2 years ago?

d. *Writing* What advice would you give to the city planners who are trying to decide whether or not to build freeways? Include in your advice a population prediction for 10 years and 20 years from now.

QUADRATIC EQUATIONS AND FUNCTIONS

What is the path of a home run ball?