CHAPTER 8

APPLICATION: Bicycle Racing

Shifting into high gear helps a racer increase speed on level or downhill surfaces, but pedaling becomes more difficult. When the racer exerts energy the brain sends a message to increase the rate and depth of breathing.

The relationship between breathing rate and bicycle speed can be represented with a type of mathematical model that you'll study in Chapter 8.

Think & Discuss

1. Construct a scatter plot of the data below. Draw a smooth curve through the points.

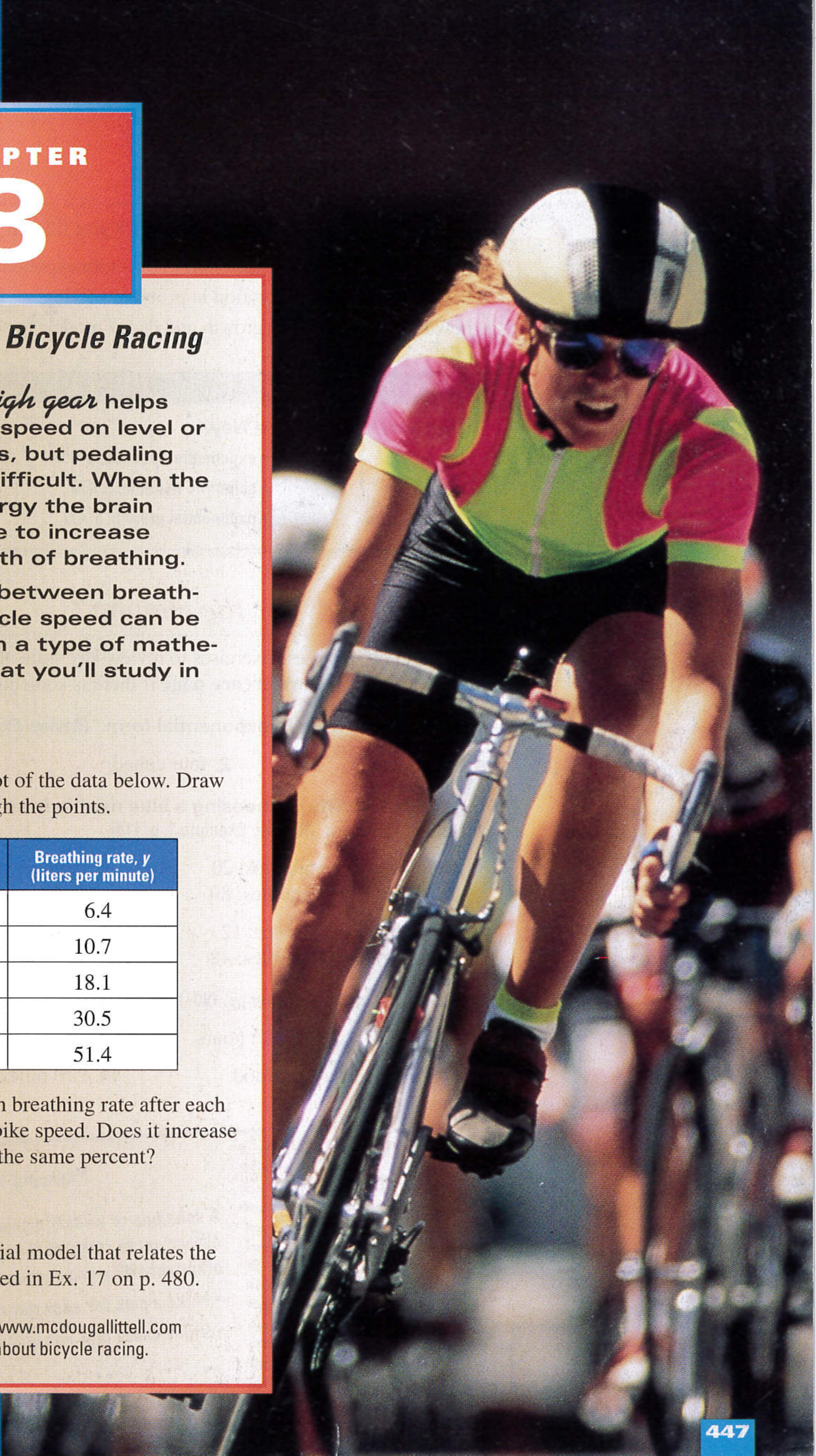
Bicycle speed, x (miles per hour)	Breathing rate, y (liters per minute)
0	6.4
5	10.7
10	18.1
15	30.5
20	51.4

2. Describe the change in breathing rate after each increase of 5 mi/h in bike speed. Does it increase by the same amount? the same percent?

Learn More About It

You will use an exponential model that relates the breathing rate to bike speed in Ex. 17 on p. 480.

APPLICATION LINK Visit www.mcdougallittell.com for more information about bicycle racing.



CHAPTER 8

Study Guide

PREVIEW

What's the chapter about?

Chapter 8 is about **exponents**. In Chapter 8 you'll learn

- how to multiply and divide expressions with exponents
- how to use scientific notation in problem solving
- how to use exponential growth and decay models to solve real-life problems

KEY VOCABULARY

▶ Review

- power, p. 9
- base, p. 9
- function, p. 46

▶ New

- exponential function, p. 458
- scientific notation, p. 470
- exponential growth, p. 477
- growth factor, p. 477
- initial amount, p. 477, 484
- exponential decay, p. 484
- decay factor, p. 484

PREPARE

Are you ready for the chapter?

SKILL REVIEW Do these exercises to review key skills that you'll apply in this chapter. See the given **reference page** if there is something you don't understand.

STUDENT HELP

Study Tip
"Student Help" boxes throughout the chapter give you study tips and tell you where to look for extra help in this book and on the Internet.

Write the expression in exponential form. (Review Example 1, p. 9)

1. six squared

2. four cubed

3. $2y \cdot 2y \cdot 2y \cdot 2y \cdot 2y$

Find the probability of choosing a blue marble from the given bag of red and blue marbles. (Review Example 1, p. 114)

4. Number of blue marbles: 20
Total number of marbles: 80

5. Number of blue marbles: 6
Total number of marbles: 54

6. Number of red marbles: 12
Total number of marbles: 48

7. Number of red marbles: 25
Total number of marbles: 50

Find the unit rate. (Review pp. 180–182)

8. \$123.75 for working 15 hours

9. \$3.16 for 4 cantaloupes

10. \$6 for 12 cans of cat food

11. 270 miles for 6 gallons of gasoline

STUDY STRATEGY

Here's a study strategy!

Planning Your Time:

A schedule or weekly planner can be a useful tool that allows you to coordinate your study time with time for other activities and responsibilities.

- Make a plan for each day of the week.
- Study every day, not just the day before a test.

Investigating Powers

SETUP

Work in a small group.

MATERIALS

- paper
- pencil

QUESTION **How can you use addition to multiply exponential expressions? How can you use multiplication to raise an exponential expression to a power?**

EXPLORING THE CONCEPT: PRODUCT OF POWERS

1. Copy and complete the table. To simplify an expression, expand the product. Then count the factors.

Product of powers	Expanded product	Number of factors	Product as a power
$7^3 \cdot 7^2$	$(7 \cdot 7 \cdot 7) \cdot (7 \cdot 7)$	5	7^5
$2^4 \cdot 2^4$	$(2 \cdot 2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2 \cdot 2)$	8	?
$x^4 \cdot x^5$	$(x \cdot x \cdot x \cdot x) \cdot (x \cdot x \cdot x \cdot x \cdot x)$	?	?

2. Add a column to your table that shows the sum of the exponents that are in the first column. What pattern do you notice?

EXPLORING THE CONCEPT: POWER OF A POWER

3. Copy and complete the table. To simplify an expression, expand the product. Then count the factors.

Power of a power	Expanded product	Expanded product	Number of factors	Product as a power
$(5^2)^3$	$(5^2) \cdot (5^2) \cdot (5^2)$	$(5 \cdot 5) \cdot (5 \cdot 5) \cdot (5 \cdot 5)$	6	5^6
$[(-3)^2]^2$	$[(-3)^2] \cdot [(-3)^2]$	?	?	?
$(b^2)^4$	?	?	?	?

4. Add a column to your table that shows the product of the exponents that are in the first column. What pattern do you notice?

DRAWING CONCLUSIONS

Expand the product. Then write your answer as a power.

1. $6^3 \cdot 6^2$ **2.** $(-2) \cdot (-2)^4$ **3.** $p^4 \cdot p^6$ **4.** $x^{12} \cdot x^7$

5. $(4^2)^6$ **6.** $[(-5)^2]^4$ **7.** $(d^5)^5$ **8.** $[(-n)^3]^8$

9. What operation do you use to simplify a product of powers? Give examples.

10. What operation do you use to simplify a power of a power? Give examples.

11. **CRITICAL THINKING** Does $x^3 \cdot y^5 = xy^8$? Explain your answer.

8.1

Multiplication Properties of Exponents

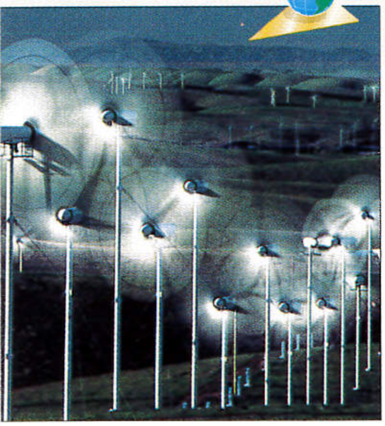
What you should learn

GOAL 1 Use properties of exponents to multiply exponential expressions.

GOAL 2 Use powers to model **real-life** problems, such as finding the area of crop irrigation circles in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems such as calculating the power generated by a windmill in **Ex. 77**.



GOAL 1 MULTIPLYING EXPONENTIAL EXPRESSIONS

To multiply two powers that have the same base, you add exponents. Here is an example.

$$a^2 \cdot a^3 = \overbrace{\underbrace{a \cdot a}_{\text{2 factors}} \cdot \underbrace{a \cdot a \cdot a}_{\text{3 factors}}}^{\text{5 factors}} = a^5 = a^{2+3}$$

To find a power of a power, you multiply exponents. Here is an example.

$$(a^2)^3 = \overbrace{a^2 \cdot a^2 \cdot a^2}^{\text{3 factors}} = \overbrace{a \cdot a \cdot a \cdot a \cdot a \cdot a}^{\text{6 factors}} = a^6 = a^{2 \cdot 3}$$

These two rules for exponents and the rule for raising a product to a power are summarized below.

CONCEPT SUMMARY MULTIPLICATION PROPERTIES OF EXPONENTS

Let a and b be numbers and let m and n be positive integers.

PRODUCT OF POWERS PROPERTY

To multiply powers having the same base, add the exponents.

$a^m \cdot a^n = a^{m+n}$ **Example:** $3^2 \cdot 3^7 = 3^{2+7} = 3^9$

POWER OF A POWER PROPERTY

To find a power of a power, multiply the exponents.

$(a^m)^n = a^{m \cdot n}$ **Example:** $(5^2)^4 = 5^{2 \cdot 4} = 5^8$

POWER OF A PRODUCT PROPERTY

To find a power of a product, find the power of each factor and multiply.

$(a \cdot b)^m = a^m \cdot b^m$ **Example:** $(2 \cdot 3)^6 = 2^6 \cdot 3^6$

EXAMPLE 1 *Using the Product of Powers Property*

a. $$\begin{aligned} 5^3 \cdot 5^6 &= 5^{3+6} \\ &= 5^9 \end{aligned}$$

b. $$\begin{aligned} x^2 \cdot x^3 \cdot x^4 &= x^{2+3+4} \\ &= x^9 \end{aligned}$$

c. $$\begin{aligned} 3 \cdot 3^5 &= 3^1 \cdot 3^5 \\ &= 3^{1+5} \\ &= 3^6 \end{aligned}$$

d. $$\begin{aligned} (-2)(-2)^4 &= (-2)^1 \cdot (-2)^4 \\ &= (-2)^{1+4} \\ &= (-2)^5 \end{aligned}$$

STUDENT HELP

Look Back
For help with exponential expressions, see page 9.

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 Using the Power of a Power Property

a. $(3^5)^2 = 3^{5 \cdot 2}$

$= 3^{10}$

b. $(y^2)^4 = y^{2 \cdot 4}$

$= y^8$

c. $[(-3)^3]^2 = (-3)^{3 \cdot 2}$

$= (-3)^6$

d. $[(a+1)^2]^5 = (a+1)^{2 \cdot 5}$

$= (a+1)^{10}$

....

When you use the power of a power property, it is the quantity within the parentheses that is raised to the power *not* the individual terms.

Correct: $(a+1)^3 = (a+1)(a+1)(a+1)$ ← **3 factors**

Incorrect: ~~$(a+1)^3 = a^3 + 1^3$~~ ← **2 terms**

EXAMPLE 3 Using the Power of a Product Property

a. $(6 \cdot 5)^2 = 6^2 \cdot 5^2$ **Raise each factor to a power.**

$= 36 \cdot 25$ **Evaluate each power.**

$= 900$ **Multiply.**

b. $(4yz)^3 = (4 \cdot y \cdot z)^3$ **Identify factors.**

$= 4^3 \cdot y^3 \cdot z^3$ **Raise each factor to a power.**

$= 64y^3z^3$ **Simplify.**

c. $(-2w)^2 = (-2 \cdot w)^2$ **Identify factors.**

$= (-2)^2 \cdot w^2$ **Raise each factor to a power.**

$= 4w^2$ **Simplify.**

d. $-(2w)^2 = -(2 \cdot w)^2$ **Identify factors.**

$= -(2^2 \cdot w^2)$ **Raise each factor to a power.**

$= -4w^2$ **Simplify.**

STUDENT HELP

Study Tip
In parts (c) and (d) of Example 3, notice that the two expressions are different. In $(-2w)^2$, the negative sign is part of the base. In $-(2w)^2$, the negative sign is not part of the base.

EXAMPLE 4 Using All Three Properties

Simplify $(4x^2y)^3 \cdot x^5$.

SOLUTION

$(4x^2y)^3 \cdot x^5 = 4^3 \cdot (x^2)^3 \cdot y^3 \cdot x^5$ **Power of a product**

$= 64 \cdot x^6 \cdot y^3 \cdot x^5$ **Power of a power**

$= 64x^{11}y^3$ **Product of powers**

GOAL 2 USING POWERS IN REAL LIFE

REAL LIFE

Irrigation

EXAMPLE 5 *Using the Power of a Product Property*

Some farmers use *center-pivot irrigation,* which is a sprinkler system that revolves around a center pivot. The large irrigation circles help farmers to conserve water, maximize crop yield, and reduce the cost of pesticides.

a. Find the ratio of the area of the larger irrigation circle to the area of the smaller irrigation circle.

b. Write a general statement about *doubling* the radius of an irrigation circle.

SOLUTION

a. Ratio $= \dfrac{\pi(2r)^2}{\pi r^2} = \dfrac{\pi \cdot 2^2 \cdot r^2}{\pi \cdot r^2} = \dfrac{\pi \cdot 4 \cdot r^2}{\pi \cdot r^2} = \dfrac{4}{1}$

b. Doubling the radius of an irrigation circle makes the area *four* times as large.

EXAMPLE 6 *Using the Product of Powers Property*

PROBABILITY CONNECTION A true-false test has two parts. There are 2^{10} ways to answer the 10 questions in Part A. There are 2^{15} ways to answer the 15 questions in Part B.

a. How many ways are there to answer all 25 questions?

b. If you guess each answer, what is the probability you will get them all right?

SOLUTION

STUDENT HELP

Skills Review
For help with the counting principle, see page 788.

a. For each of the 2^{10} ways to answer the questions in Part A, there are 2^{15} ways to answer the questions in Part B. Use the counting principle to find the total number of ways to answer for both parts. The number of ways to answer the 25 questions is the product of 2^{10} and 2^{15}.

$$2^{10} \cdot 2^{15} = 2^{10+15} \quad \text{Use product of powers property.}$$

$$= 2^{25} \quad \text{Add exponents.}$$

$$= 33{,}554{,}432 \quad \text{Use a calculator.}$$

▶ The number of ways to answer the 25 questions is 33,554,432.

b. Probability $= \dfrac{\text{Ways to get all right}}{\text{Ways to answer}} = \dfrac{1}{33{,}554{,}432}$

▶ The probability of guessing and getting all answers correct is about 0.00000003.

GUIDED PRACTICE

Vocabulary Check ✓

1. In the expression a^5, a is called the _?_ of the expression.

Concept Check ✓

2. How are the expressions $x^7 \cdot x^3$ and $(x^7)^3$ different? Explain your answer.
3. Can $a^3 \cdot b^4$ be simplified? Explain your answer.

Skill Check ✓

Use the product of powers property to simplify the expression.

4. $c \cdot c \cdot c$
5. $m \cdot m^2$
6. $2^2 \cdot 2^3$
7. $3^2 \cdot 3^5$
8. $a^4 \cdot a^6$
9. $x^4 \cdot x^5$

Use the power of a power property to simplify the expression.

10. $(3)^2$
11. $(-2)^2$
12. $(2^4)^3$
13. $(4^3)^3$
14. $(y^4)^5$
15. $(m^4)^8$

Use the power of a product property to simplify the expression.

16. $(2m^2)^3$
17. $(ab^2)^2$
18. $(5x)^2$
19. $(x^3y^5)^4$
20. $(x^3y^8)^5$
21. $(-2x^3)^3$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

SIMPLIFYING EXPRESSIONS **Simplify, if possible. Write your answer as a power or as a product of powers.**

22. $3^4 \cdot 3^6$
23. $5^8 \cdot 5^3$
24. $(2^3)^2$
25. $(7^4)^2$
26. $x \cdot x^6$
27. $(3 \cdot 7)^2$
28. $(2x)^2$
29. $(-5a)^3$
30. $(-2m^4n^6)^2$
31. $[(-4)^2]^3$
32. $[(-5xy)^2]^5$
33. $[(5+x)^3]^6$
34. $[(2x+3)^3]^2$
35. $(3b)^3 \cdot b$
36. $5^3 \cdot (5a^4)^2$
37. $4x \cdot (x \cdot x^3)^2$
38. $(-3a)^5 \cdot (4a)^2$
39. $-(3x)^2 \cdot (7x^4)^2$
40. $2x^3 \cdot (3x)^2$
41. $3y^2 \cdot (2y)^3$
42. $(-ab)(a^2b)^2$
43. $(-rs)(rs^3)^2$
44. $(-2xy)^3(-x^2)$
45. $(-3cd)^3(-d^2)$
46. $(5b^2)^3\left(\frac{1}{2}b^3\right)^2$
47. $(6a^4)^2\left(\frac{1}{4}a^3\right)^2$
48. $(2t)^3(-t^2)$
49. $(-w^3)(3w^2)^2$
50. $(-y)^3(-y)^4(-y)^5$
51. $(-x)^4(-x)^3(-x)^2$
52. $(abc^2)^3(a^2b)^2$
53. $-(r^2st^3)^2(s^4t)^3$
54. $(-3xy^2)^3(-2x^2y)^2$

EVALUATING EXPRESSIONS **Simplify. Then evaluate the expression when $a = 1$ and $b = 2$.**

55. $a^2 \cdot a^3$
56. $b \cdot b^4$
57. $(a^3)^2$
58. $(-b)^3 \cdot b^2$
59. $(a \cdot b^2)^2$
60. $(a^2b)^4$
61. $-(ab^3)^2$
62. $(b^2 \cdot b^3) \cdot (b^2)^4$

STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 22–62
Example 2: Exs. 22–62
Example 3: Exs. 22–62
Example 4: Exs. 36–62
Example 5: Exs. 77, 78
Example 6: Ex. 79

WRITING INEQUALITIES **Complete the statement using > or <.**

63. $(5 \cdot 6)^4 \underline{\ ?\ } 5 \cdot 6^4$

64. $5^2 \cdot 3^6 \underline{\ ?\ } (5 \cdot 3)^6$

65. $(3^6 \cdot 3^{12}) \underline{\ ?\ } 3^{72}$

66. $4^2 \cdot 4^8 \underline{\ ?\ } 4^{16}$

67. $(7^2)^3 \underline{\ ?\ } 7^5$

68. $(6^2 \cdot 3)^3 \underline{\ ?\ } 6^5 \cdot 3^3$

EVALUATING POWERS **In Exercises 69–74, simplify the expression. Then use a calculator to evaluate the expression. Round the result to the nearest tenth when appropriate.**

69. $(2.1 \cdot 4.4)^3$

70. $6.5^3 \cdot 6.5^4$

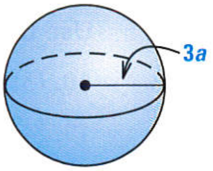
71. $2.6^4 \cdot 2.6^2$

72. $(5.0 \cdot 4.9)^2$

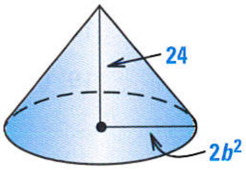
73. $(3.7^3)^5$

74. $(8.4^2)^4$

75. **GEOMETRY CONNECTION** The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius and π is approximately 3.14. What is the volume of the sphere in terms of a?



76. **GEOMETRY CONNECTION** The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$, where r is the radius of the base, h is the height, and $\pi \approx 3.14$. What is the volume of the cone in terms of b?



WINDMILLS **In Exercises 77 and 78, use the following information. The power generated by a windmill can be modeled by the equation $w = 0.015s^3$, where w is the power measured in watts and s is the wind speed in miles per hour.**

77. Find the ratio of the power generated by a windmill when the wind speed is 20 miles per hour to the power generated when the wind speed is 10 miles per hour.

78. *Writing* Write a general statement about how doubling the wind speed affects the amount of power generated by a windmill.

79. **PROBABILITY CONNECTION** Part A of a test has 10 true-false questions. Part B has 10 multiple-choice questions. Each of the multiple-choice questions has 4 possible answers. There are 2^{10} ways to answer the 10 questions in Part A. There are 4^{10} ways to answer the 10 questions in Part B.

a. How many ways are there to answer all 20 questions?

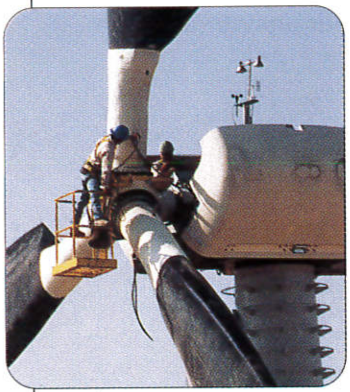
b. If you guess the answer to each question, what is the probability that you will get them all right?

PROBABILITY CONNECTION **In Exercises 80 and 81, suppose you put one red marble, one green marble, and one blue marble in each of six bags. There are 3^6 possible orderings of the colors of the marbles you can get when you choose one marble from each bag.**

80. If you use 8 bags, there are 3^8 possible orderings of colors. What is the probability that the marbles you choose will all be red?

81. If you use 14 bags, how many different orderings of colors are there? What is the probability that the marbles you choose will all be red?

FOCUS ON CAREERS



ENGINEERING TECHNICIANS use science, engineering, and math to solve technical problems in the development, construction, inspection, and maintenance of industrial machinery.

CAREER LINK
www.mcdougallittell.com

Test Preparation

MULTI-STEP PROBLEM **Use the results of Exercise 82 for Exercise 83.**

82. a. Copy and complete the table of values.

x	0	1	2	3	4
$2x$	?	?	?	?	?
2^x	?	?	?	?	?

b. Sketch the graphs of $y = 2x$ and $y = 2^x$ in the same coordinate plane.

c. Compare the graphs. How are they the same? How are they different?

83. CRITICAL THINKING You are offered a job that pays $2x$ dollars or 2^x dollars for x hours of work. Assuming you must work at least 2 hours, which method of payment would you choose? Explain your reasoning.

★ Challenge

84. LOGICAL REASONING Fill in the blanks and give a reason for each step to complete a convincing argument that the power of a power property is true.

$$\begin{aligned}(a^2)^3 &= a^2 \cdot \underline{?} \cdot \underline{?} \\ &= \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?} \cdot \underline{?} \\ &= \underline{?}\end{aligned}$$

EXTRA CHALLENGE
www.mcdougallittell.com

85. LOGICAL REASONING Write a convincing argument to show that the power of a product property is true.

MIXED REVIEW

EXPONENTIAL EXPRESSIONS **Evaluate the expression.** **(Review 1.3 for 8.2)**

86. b^2 when $b = 8$

87. $(5y)^4$ when $y = 2$

88. $\frac{1}{2}n^3$ when $n = -2$

89. $\frac{1}{y^2}$ when $y = 5$

90. $\frac{24}{x^3}$ when $x = 2$

91. $\frac{45}{a^2}$ when $a = 2$

GRAPHING EQUATIONS **Use a table of values to graph the equation.** **(Review 4.2)**

92. $y = x + 2$

93. $y = -(x - 4)$

94. $y = \frac{1}{2}x - 5$

95. $y = \frac{3}{4}x + 2$

96. $y = 2$

97. $x = -3$

GRAPHING INEQUALITIES **Sketch the graph of the inequality.** **(Review 6.1)**

98. $x < 4$

99. $x > 15$

100. $x \geq -9$

101. $x \leq 3$

SOLVING INEQUALITIES **Solve the inequality.** **(Review 6.2)**

102. $-x - 2 < -5$

103. $8 + 3x \geq -2$

104. $2 < 2x + 7$

105. PRICE OF MILK In 1910, a quart of milk cost \$.07. In 1994, a quart of milk cost \$.71. After 1910, the price of milk increased steadily, never falling below \$.07 per quart again. During the period 1910–1994, the maximum price of a quart was \$.71. Write a compound inequality that represents the possible costs of a quart of milk between 1910 and 1994. **(Review 6.3)**

8.2

Zero and Negative Exponents

What you should learn

GOAL 1 Evaluate powers that have zero and negative exponents.

GOAL 2 Graph exponential functions.

Why you should learn it

▼ To solve **real-life** problems such as predicting a player's average score per game in **Example 6**.

REAL LIFE

GOAL 1 USING ZERO AND NEGATIVE EXPONENTS

In this lesson you will study how to use the multiplication properties of exponents when working with negative exponents.

ACTIVITY Developing Concepts

Investigating Zero and Negative Exponents

① Copy the table and discuss any patterns you see. Use the patterns to complete the table. Write non-integers as fractions in simplest form.

Exponent, n	3	2	1	0	-1	-2	-3
Power, 2^n	8	4	2	?	?	?	?
Power, 3^n	27	9	3	?	?	?	?
Power, 4^n	64	16	4	?	?	?	?

② What appears to be the value of a^0 for any number a?

③ How can you evaluate an expression of the form a^{-n}?

DEFINITION OF ZERO AND NEGATIVE EXPONENTS

Let a be a nonzero number and let n be a positive integer.

- A nonzero number to the zero power is 1: $a^0 = 1, a \neq 0$.
- a^{-n} is the reciprocal of a^n: $a^{-n} = \frac{1}{a^n}, a \neq 0$.

EXAMPLE 1 *Powers with Zero and Negative Exponents*

a. $2^{-2} = \frac{1}{2^2} = \frac{1}{4}$ — 2^{-2} is the reciprocal of 2^2.

b. $(-2)^0 = 1$ — a^0 is 1.

c. $5^{-x} = \frac{1}{5^x}$ — 5^{-x} is the reciprocal of 5^x.

d. $\left(\frac{1}{3}\right)^{-1} = 3$ — The reciprocal of $\frac{1}{3}$ is 3.

e. $0^{-3} = \cancel{\frac{1}{0^3}}$ (Undefined) — Zero has no reciprocal.

STUDENT HELP

HOMEWORK HELP Visit our Web site www.mcdougallittell.com for extra examples.

EXAMPLE 2 Simplifying Exponential Expressions

Rewrite with positive exponents.

a. $5(2^{-x})$ **b.** $2x^{-2}y^{-3}$

SOLUTION

a. $5(2^{-x}) = 5\left(\frac{1}{2^x}\right) = \frac{5}{2^x}$

b. $2x^{-2}y^{-3} = 2 \cdot \frac{1}{x^2} \cdot \frac{1}{y^3} = \frac{2}{x^2y^3}$

STUDENT HELP

Study Tip

Informally, you can think of rewriting expressions with positive exponents as "moving factors" from the denominator to the numerator or vice versa.

$$\frac{1}{3x^{-4}} = \frac{x^4}{3}$$

EXAMPLE 3 Evaluating Exponential Expressions

Evaluate the expression.

a. $3^{-2} \cdot 3^2$ **b.** $(2^{-3})^{-2}$ **c.** 3^{-4}

SOLUTION

a. $3^{-2} \cdot 3^2 = 3^{-2+2}$ **Use product of powers property.**

$= 3^0$ **Add exponents.**

$= 1$ **a^0 is 1.**

b. $(2^{-3})^{-2} = 2^{-3 \cdot (-2)}$ **Use power of a power property.**

$= 2^6$ **Multiply exponents.**

$= 64$ **Evaluate.**

c. You might want to evaluate 3^{-4} with a calculator.

KEYSTROKES	DISPLAY
3 y^x 4 +/- =	.01234568

STUDENT HELP

KEYSTROKE HELP

Use y^x or ^ to input the exponent −4.

EXAMPLE 4 Simplifying Exponential Expressions

Rewrite with positive exponents.

a. $(5a)^{-2}$ **b.** $\frac{1}{d^{-3n}}$

SOLUTION

a. $(5a)^{-2} = 5^{-2} \cdot a^{-2}$ **Use power of a product property**

$= \frac{1}{5^2} \cdot \frac{1}{a^2}$ **Write reciprocals of 5^2 and a^2.**

$= \frac{1}{25a^2}$ **Multiply fractions.**

b. $\frac{1}{d^{-3n}} = (d^{-3n})^{-1}$ **Use definition of negative exponents.**

$= d^{(-3n) \cdot (-1)}$ **Use power of a power property.**

$= d^{3n}$ **Multiply exponents.**

GOAL 2 GRAPHING EXPONENTIAL FUNCTIONS

So far we have used expressions of the form b^n where $b \neq 0$ and n is an integer. To model some situations, we need an **exponential function** of the form $y = a \cdot b^x$ where x is a real number, $b > 0$, and $b \neq 1$. To graph the exponential function $y = a \cdot b^x$, make a table using integer values for x and plot the corresponding points. To complete the graph for all real values of x, connect the points with a smooth curve.

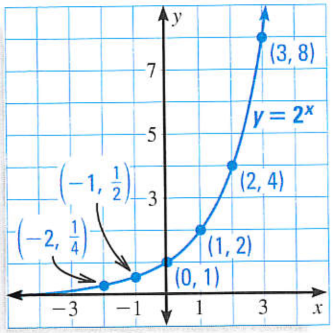
EXAMPLE 5 *Graphing an Exponential Function*

To sketch the graph of $y = 2^x$, make a table that includes negative x-values.

x	-2	-1	0	1	2	3
2^x	$2^{-2} = \frac{1}{4}$	$2^{-1} = \frac{1}{2}$	$2^0 = 1$	2	4	8

Draw a coordinate plane and plot the six points given by the table. Then draw a smooth curve through the points.

Notice that the graph has a y-intercept of 1, and that it gets closer to the negative side of the x-axis as the x-values get smaller.



In real-life applications of $y = ab^x$, x is often the time period.

REAL LIFE
Basketball

EXAMPLE 6 *Evaluating an Exponential Function*

A professional basketball player's first year in the NBA was 1998. Suppose it were estimated that from 1998 to 2010 his average points per game could be modeled by $P = 18.5(1.038)^t$, where $t = 0$ represents the year 2000.

a. Estimate the player's average points per game in 1998.

b. Estimate his average points per game in the year 2000.

SOLUTION

a. $P = 18.5 \cdot 1.038^{-2}$ **Substitute −2 for *t*.**

≈ 17.17 **Use a calculator.**

▶ In the year 1998 his average points per game was about 17.2 points.

b. $P = 18.5 \cdot 1.038^{0}$ **Substitute 0 for *t*.**

$= 18.5$ **a^0 is 1.**

▶ In the year 2000 his average points per game was about 18.5 points.

GUIDED PRACTICE

Vocabulary Check ✓

1. The function $y = ab^x$ is a(n) _?_ function.

Concept Check ✓

2. ERROR ANALYSIS Describe the error at the right.

$$\cancel{5x^{-3} = \frac{1}{5x^3}}$$

Skill Check ✓

Evaluate the expression.

3. 3^{-1} **4.** 0^{-4} **5.** 0^0 **6.** $6 \cdot 3^0$

Rewrite as an expression with positive exponents.

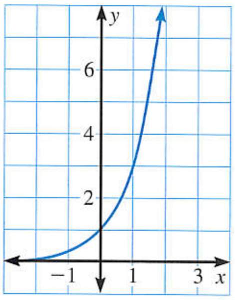
7. m^{-2} **8.** $3c^{-5}$ **9.** a^5b^{-8} **10.** $\frac{1}{(2x)^{-3}}$

11. Does the graph at the right appear to be the graph of the exponential function $y = 3^x$?

12. Tell whether the following statement is true.

If a is positive, then a^{-n} is positive.

Explain your reasoning.



Ex. 11

13. BASKETBALL Use the information in Example 6. If the player's final year in the NBA is 2010, estimate his average points per game in his final year.

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

EVALUATING EXPRESSIONS **Evaluate the exponential expression. Write fractions in simplest form.**

14. 4^{-2} **15.** 3^{-4} **16.** $\left(\frac{1}{5}\right)^{-1}$ **17.** $8\left(\frac{1}{4}\right)^{-1}$

18. $4(4^{-2})$ **19.** $\left(\frac{1}{10}\right)^{-2}$ **20.** $-6^0 \cdot \frac{1}{3^{-2}}$ **21.** $2^{-3} \cdot 2^2$

22. $8^3 \cdot 0^{-1}$ **23.** $7^4 \cdot 7^{-4}$ **24.** $8^{-7} \cdot 8^7$ **25.** $-4 \cdot (-4)^{-1}$

26. $(5^{-3})^2$ **27.** $(-3^{-2})^{-1}$ **28.** $11 \cdot 11^{-1}$ **29.** $4^0 \cdot 5^{-3}$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 14–29
Example 2: Exs. 30–45
Example 3: Exs. 14–29, 49–52
Example 4: Exs. 30–45
Example 5: Exs. 53–63
Example 6: Exs. 64, 65

SIMPLIFYING EXPRESSIONS **Rewrite the expression with positive exponents.**

30. x^{-5} **31.** $3x^{-4}$ **32.** $\frac{1}{2x^{-5}}$ **33.** $x^{-2}y^4$

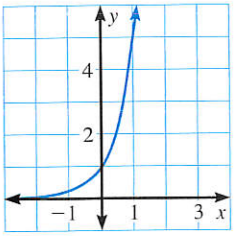
34. x^4y^{-7} **35.** $8x^{-2}y^{-6}$ **36.** $\frac{1}{9x^{-3}y^{-1}}$ **37.** $\frac{1}{4x^{-10}y^{14}}$

38. $(-9)^0x$ **39.** $(-4x)^{-3}$ **40.** $(-10a)^0$ **41.** $(3xy)^{-2}$

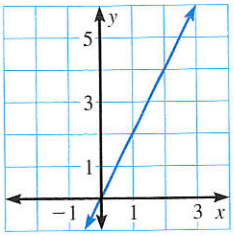
42. $(6a^{-3})^3$ **43.** $\frac{8}{m^{-2}}$ **44.** $\frac{1}{(4x)^{-5}}$ **45.** $\left(\frac{-4x^2}{2x^{-1}}\right)^{-1}$

MATCHING THE GRAPH Match the equation with its graph.

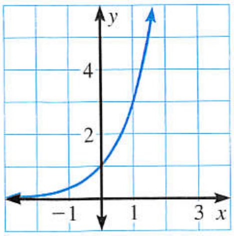
A.



B.



C.



46. $y = 2x$ **47.** $y = 3^x$ **48.** $y = 5^x$

EVALUATING EXPONENTIAL EXPRESSIONS **Use a calculator to evaluate the expression. Round your answer to the nearest hundred thousandth.**

49. 2^{-5} **50.** $(1.1)^{-2}$ **51.** $55 \cdot 5^{-6}$ **52.** $3^{-5} \div 0.9$

CHECKING POINTS **Does the graph of the function contain the point (0, 1)?**

53. $y = -3^x$ **54.** $y = 4^x$ **55.** $y = 4 \cdot 1^x$ **56.** $y = 60^x$

GRAPHING FUNCTIONS **In Exercises 57–60, graph the exponential function.**

57. $y = \left(\frac{1}{3}\right)^x$ **58.** $y = \left(\frac{1}{5}\right)^x$ **59.** $y = 4^{-x}$ **60.** $y = 5^x$

61. VISUAL THINKING Sketch the graphs of $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$. How are the graphs related?

62. CRITICAL THINKING Sketch the graphs of $y = 3^x$ and $y = \left(\frac{1}{3}\right)^x$. Use these graphs and the ones you sketched in Exercise 61 to predict how the graphs of $y = b^x$ and $y = \left(\frac{1}{b}\right)^x$ are related.

63. COMMON POINTS What point do all graphs of the form $y = a^x$ have in common? Is there a point that all graphs of the form $y = 2(a)^x$ have in common? If so, name the point.

64. SAVINGS ACCOUNT You started a savings account in 1990. The balance A is modeled by $A = 450(1.06)^t$, where $t = 0$ represents the year 2000. What is the balance in the account in 1990? in 2000? in 2010?

65. SHIPWRECKS Suppose that from 1860 to 1980 the number of shipwrecks in the Gulf of Mexico increased by about the same percent each year and that the number of shipwrecks S for each decade t can be modeled by $S = 292(1.2)^t$, where $t = 0$ represents the decade 1920 to 1929.

a. Copy and complete the table below.

	1870–1879	1880–1889	1900–1909	1910–1919	1920–1929
t	−5	−4	−2	−1	0
S	?	?	?	?	?

b. Graph the function and check your results.

FOCUS ON APPLICATIONS

REAL LIFE **SHIPWRECKS** In 1996 the excavation of the *Belle* off the coast of Texas uncovered the hull of the ship, three bronze cannons, millions of glass beads, pottery and even the skeleton of a crew member.

INTERNET **APPLICATION LINK**
www.mcdougallittell.com

Test Preparation

QUANTITATIVE COMPARISON **In Exercises 66–68, evaluate each function. Then choose the statement below that is true about the given values of y.**

Ⓐ The value of y in column A is greater.

Ⓑ The value of y in column B is greater.

Ⓒ The two values of y are equal.

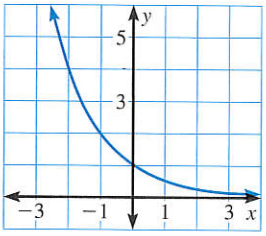
Ⓓ The relationship cannot be determined from the given information.

		Column A	Column B
66.	When $x = 3$,	$y = 2x$	$y = 2^x$
67.	When $x = 1$,	$y = 2^x$	$y = 2^{-x}$
68.	When $x = 0$,	$y = \left(\frac{1}{2}\right)^x$	$y = \left(\frac{1}{2}\right)^{-x}$

69. **MULTIPLE CHOICE** What is a possible equation of the graph?

Ⓐ $y = 2^x$ Ⓑ $y = 3^x$

Ⓒ $y = \left(\frac{1}{2}\right)^x$ Ⓓ $y = \left(\frac{1}{3}\right)^x$



★ Challenge

70. *Writing* Suppose you did not know that for $b \neq 0$, $b^0 = 1$. Based on the equation $b^2 \cdot b^0 = b^{2+0} = b^2$, explain why you might want to make this definition.

MIXED REVIEW

EVALUATING EXPRESSIONS **Evaluate the expression.** **(Review 1.3 for 8.3)**

71. $\left(\frac{2}{5}\right)^2$ **72.** $\left(\frac{1}{2}\right)^3$ **73.** $\left(-\frac{9}{10}\right)^3$ **74.** $\left(\frac{1}{5}\right)^4$

SOLVE AND GRAPH **Solve the inequality. Then sketch a graph of the solution on a number line.** **(Review 6.4)**

75. $|5 + x| + 4 \leq 11$ **76.** $|3x + 7| - 4 > 9$ **77.** $|x + 2| - 1 \leq 8$

78. $|3 - x| - 6 > -4$ **79.** $|9 - 2x| + 3 < 4$ **80.** $|3x + 2| + 9 \geq -1$

STATISTICS **Draw a box-and-whisker plot of the data.** **(Review 6.7)**

81. 48, 10, 48, 25, 40, 42, 44, 23, 21, 13, 50, 17

82. 85, 61, 55, 78, 79, 86, 30, 76, 76, 87, 68, 82

SOLVING SYSTEMS **Use substitution to solve the system.** **(Review 7.2)**

83. $2x - y = -2$
$4x + y = 5$

84. $-3x + y = 4$
$-9x + 5y = 10$

85. $x + 4y = 300$
$x - 2y = 0$

86. $2x - 3y = 10$
$3x + 3y = 15$

87. $x + 15y = 6$
$-x - 5y = 84$

88. $4x - y = 5$
$2x + 4y = 15$

ACTIVITY 8.2

Using Technology

Graphing Exponential Functions

You can use a graphing calculator to graph an exponential function.

EXAMPLE

Graph $y = \left(\frac{1}{2}\right)^x$.

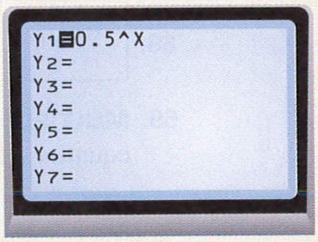
SOLUTION

STUDENT HELP

INTERNET KEYSTROKE HELP

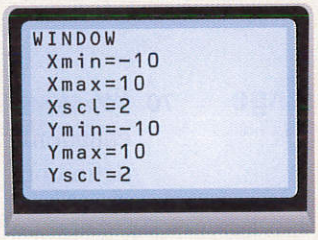
See keystrokes for several models of calculators at www.mcdougallittell.com

① To enter the function in your graphing calculator, press Y= .
Enter the function as
0 . 5 ^ X, T, θ .



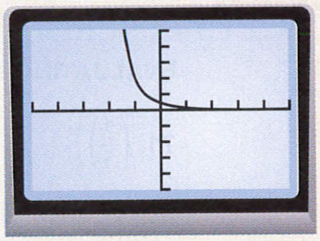
```
Y1=0.5^X
Y2=
Y3=
Y4=
Y5=
Y6=
Y7=
```

② Adjust the *viewing window* to get the best scale for your graph.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=2
 Ymin=-10
 Ymax=10
 Yscl=2
```

③ Now you are ready to graph the function. Press GRAPH to see the graph.



EXERCISES

Use a graphing calculator to graph the exponential function.

1. $y = 2^x$

2. $y = 10^x$

3. $y = -3^x$

4. $y = 5^{-x}$

5. $y = (0.27)^x$

6. $y = -\left(\frac{2}{3}\right)^x$

CRITICAL THINKING **Use your results from Exercises 1–6 to answer the following questions.**

7. If $a > 1$, what does the graph of $y = a^x$ look like?

8. If $0 < a < 1$, what does the graph of $y = a^x$ look like?

9. If $a > 1$, what does the graph of $y = -(a^x)$ look like?

10. If $0 < a < 1$, what does the graph of $y = -(a^x)$ look like?

8.3

Division Properties of Exponents

What you should learn

GOAL 1 Use the division properties of exponents to evaluate powers and simplify expressions.

GOAL 2 Use the division properties of exponents to find a probability as in **Example 5**.

Why you should learn it

▼ To solve **real-life** problems such as comparing the top speeds of boats in an Olympic rowing competition in **Ex. 58**.

GOAL 1 DIVIDING WITH EXPONENTS

In Lesson 8.1 you learned that you multiply powers with the same base by adding exponents. To divide powers with the same base, you subtract exponents. Here is an example.

$$\frac{4^{5}}{4^{3}} = \frac{\overbrace{4 \cdot 4 \cdot 4 \cdot 4 \cdot 4}^{5\text{ factors}}}{\underbrace{4 \cdot 4 \cdot 4}_{3\text{ factors}}} = \underbrace{4 \cdot 4}_{2\text{ factors}} = 4^{5-3} = 4^{2}$$

CONCEPT SUMMARY **DIVISION PROPERTIES OF EXPONENTS**

Let a and b be numbers and let m and n be integers.

QUOTIENT OF POWERS PROPERTY

To divide powers having the same base, subtract exponents.

$$\frac{a^m}{a^n} = a^{m-n},\ a \neq 0 \qquad \textbf{Example: } \frac{3^7}{3^5} = 3^{7-5} = 3^2$$

POWER OF A QUOTIENT PROPERTY

To find a power of a quotient, find the power of the numerator and the power of the denominator and divide.

$$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m},\ b \neq 0 \qquad \textbf{Example: } \left(\frac{4}{5}\right)^3 = \frac{4^3}{5^3}$$

EXAMPLE 1 *Using the Quotient of Powers Property*

a. $\frac{6^5}{6^4} = 6^{5-4}$

$= 6^1$

$= 6$

b. $\frac{(-5)^2}{(-5)^2} = (-5)^{2-2}$

$= (-5)^0$

$= 1$

c. $\frac{9^4 \cdot 9^2}{9^7} = \frac{9^6}{9^7}$

$= 9^{6-7}$

$= 9^{-1}$

$= \frac{1}{9}$

d. $\frac{1}{y^5} \cdot y^3 = \frac{y^3}{y^5}$

$= y^{3-5}$

$= y^{-2}$

$= \frac{1}{y^2}$

EXAMPLE 2 Using the Power of a Quotient Property

STUDENT HELP

HOMEWORK HELP
Visit our Web site
www.mcdougallittell.com
for extra examples.

Simplify the expression.

a. $\left(\frac{2}{3}\right)^2$ **b.** $\left(-\frac{3}{y}\right)^3$ **c.** $\left(\frac{7}{4}\right)^{-3}$

SOLUTION

a. $\left(\frac{2}{3}\right)^2 = \frac{2^2}{3^2} = \frac{4}{9}$ Square numerator and denominator and simplify.

b. $\left(-\frac{3}{y}\right)^3 = \left(\frac{-3}{y}\right)^3$ Rewrite fraction.

$= \frac{(-3)^3}{y^3}$ Power of a quotient

$= \frac{-27}{y^3}$ Simplify.

c. $\left(\frac{7}{4}\right)^{-3} = \frac{7^{-3}}{4^{-3}}$ Power of a quotient

$= \frac{4^3}{7^3}$ Definition of negative exponents

$= \frac{64}{343}$ Simplify.

EXAMPLE 3 Simplifying Expressions

Simplify the expression.

a. $\frac{2x^2y}{3x} \cdot \frac{9xy^2}{y^4}$ **b.** $\left(\frac{2x}{y^2}\right)^4$

SOLUTION

a. $\frac{2x^2y}{3x} \cdot \frac{9xy^2}{y^4} = \frac{(2x^2y)(9xy^2)}{(3x)(y^4)}$ Multiply fractions.

$= \frac{18x^3y^3}{3xy^4}$ Product of powers

$= 6x^2y^{-1}$ Quotient of powers

$= \frac{6x^2}{y}$ Definition of negative exponents

b. $\left(\frac{2x}{y^2}\right)^4 = \frac{(2x)^4}{(y^2)^4}$ Power of a quotient

$= \frac{2^4 \cdot x^4}{y^{2 \cdot 4}}$ Product of a product and power of a power

$= \frac{16x^4}{y^8}$ Simplify.

FOCUS ON CAREERS

FLOOR TRADER Floor traders at the New York Stock Exchange use hand signals that date back to the 1880s to relay information about their trades.

CAREER LINK
www.mcdougallittell.com

GOAL 2 USING POWERS AS REAL-LIFE MODELS

EXAMPLE 4 *Using the Quotient of Powers Property*

STOCK EXCHANGE The number of shares N (in billions) listed on the New York Stock Exchange from 1977 through 1997 can be modeled by

$$N = 92.56 \cdot (1.112)^t$$

where $t = 0$ represents 1990. Find the ratio of shares listed in 1997 to the shares listed in 1977. ▶ Source: Federal Reserve Bank of New York

SOLUTION

Use $t = -13$ for 1977 and $t = 7$ for 1997.

$$\frac{\text{Number listed in 1997}}{\text{Number listed in 1977}} = \frac{92.56 \cdot (1.112)^7}{92.56 \cdot (1.112)^{-13}}$$

$$= 1.112^{7-(-13)}$$

$$= 1.112^{20}$$

$$\approx 8.4$$

▶ The ratio of shares listed in 1997 to the shares listed in 1977 is 8.4 to 1. There were about 8.4 times as many listed in 1997 as in 1977.

EXAMPLE 5 *Using the Power of a Quotient Property*

PROBABILITY CONNECTION You toss a fair coin ten times. Show that the probability that the coin lands heads up each time is about 0.001.

SOLUTION

Probability that the first toss is heads: $\frac{1}{2}$

Probability that the first two tosses are heads: $\left(\frac{1}{2}\right)^2$

Probability that the first three tosses are heads: $\left(\frac{1}{2}\right)^3$

⋮ ⋮

Probability that the first nine tosses are heads: $\left(\frac{1}{2}\right)^9$

Probability that the first ten tosses are heads: $\left(\frac{1}{2}\right)^{10}$

Use the power of a quotient property to evaluate.

$$\left(\frac{1}{2}\right)^{10} = \frac{1}{2^{10}} = \frac{1}{1024} \approx 0.001$$

▶ The probability is $\left(\frac{1}{2}\right)^{10}$ or about 0.001.

STUDENT HELP

Look Back
For help with probability, see page 114.

GUIDED PRACTICE

Vocabulary Check ✓ 1. The expression $\frac{a^4}{a^6}$ can be simplified by using the $\underline{?}$ property.

Concept Check ✓ 2. Can $\frac{x^8}{y^3}$ be simplified? Explain your answer.

Skill Check ✓ **Use the quotient of powers property to simplify the expression.**

3. $\frac{5^4}{5^1}$

4. $\frac{7^6}{7^9}$

5. $\frac{a^{12}}{a^9}$

6. $\frac{m^5}{m^{11}}$

7. $\frac{a^5}{a^2}$

8. $\frac{(-2)^8}{(-2)^3}$

9. $\frac{5^3 \cdot 5^5}{5^9}$

10. $\frac{x^7 \cdot x}{x^{-2}}$

Use the power of a quotient property to simplify the expression.

11. $\left(\frac{1}{2}\right)^5$

12. $\left(\frac{3}{5}\right)^3$

13. $\left(\frac{5}{m}\right)^2$

14. $\left(\frac{2}{b}\right)^4$

15. $\left(\frac{5}{4}\right)^{-3}$

16. $\left(\frac{x^4}{2^3}\right)^2$

17. $\left(\frac{x^3}{y^5}\right)^6$

18. $\left(\frac{a^6}{b^9}\right)^5$

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice
to help you master
skills is on p. 804.

EVALUATING EXPRESSIONS **Evaluate the expression. Write fractions in simplest form.**

19. $\frac{5^6}{5^3}$

20. $\frac{8^3}{8^1}$

21. $\frac{(-3)^6}{-3^6}$

22. $\frac{(-3)^9}{(-3)^9}$

23. $\frac{3^3}{3^{-4}}$

24. $\frac{8^3 \cdot 8^2}{8^5}$

25. $\frac{5 \cdot 5^4}{5^8}$

26. $\left(\frac{3}{4}\right)^2$

27. $\left(\frac{6}{2}\right)^3$

28. $\left(-\frac{2}{3}\right)^3$

29. $\left(-\frac{3}{5}\right)^2$

30. $\left(\frac{9}{6}\right)^{-1}$

SIMPLIFYING EXPRESSIONS **Simplify the expression. The simplified expression should have no negative exponents.**

31. $\left(\frac{3}{x}\right)^4$

32. $\frac{x^4}{x^5}$

33. $\left(\frac{1}{x}\right)^5$

34. $x^3 \cdot \frac{1}{x^2}$

35. $x^5 \cdot \frac{1}{x^8}$

36. $\left(\frac{a^9}{a^5}\right)^{-1}$

37. $\left(\frac{y^2}{y^3}\right)^{-2}$

38. $\frac{m^3 \cdot m^5}{m^2}$

39. $\frac{(r^3)^4}{(r^3)^8}$

40. $\left(\frac{-6x^2y}{2xy^3}\right)^3$

41. $\left(\frac{2x^3y^4}{3xy}\right)^3$

42. $\frac{16x^3y}{-4xy^3} \cdot -\frac{2xy}{-x^{-1}}$

43. $\frac{4x^3y^3}{2xy} \cdot \frac{5xy^2}{2y}$

44. $\frac{36a^8b^2}{ab} \cdot \left(\frac{6}{ab^2}\right)^{-1}$

45. $\frac{16x^5y^{-8}}{x^7y^4} \cdot \left(\frac{x^3y^2}{8xy}\right)^4$

46. $\frac{6x^{-2}y^2}{xy^{-3}} \cdot \frac{(4x^2y)^{-2}}{xy^2}$

47. $\frac{5x^{-3}y^2}{x^5y^{-1}} \cdot \frac{(2xy^3)^{-2}}{xy}$

48. $\left(\frac{2xy^{-2}y^4}{3x^{-1}y}\right)^{-2} \cdot \left(\frac{4xy}{2x^{-1}y^{-3}}\right)^2$

STUDENT HELP

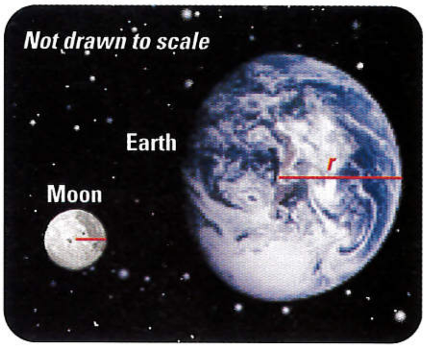
HOMEWORK HELP
Example 1: Exs. 19–50
Example 2: Exs. 19–48
Example 3: Exs. 31–50
Example 4: Exs. 51–59
Example 5: Exs. 60, 61

ERROR ANALYSIS **In Exercises 49 and 50, find and correct the errors.**

49.
$$6^3 \div 6 = \frac{6^3}{6} = 1^3 = 1$$

50.
$$\frac{x^{-9}}{x^{-3}} = x^{-9-3} = x^{-12} = \frac{1}{x^{-12}}$$

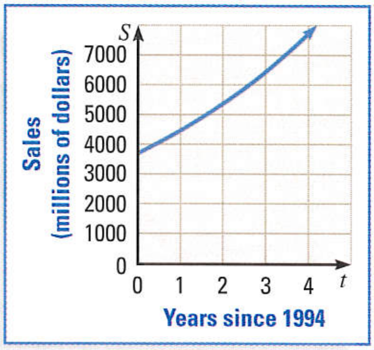
51. **EARTH AND MOON** The volume of a sphere is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the sphere. Assuming that the radius of the Moon is $\frac{1}{4}$ the radius of Earth, find the ratio of the volume of Earth to the volume of the Moon. Let r represent the radius of Earth.



52. **RETAIL SALES** From 1994 to 1998, the sales for a national clothing store increased by about the same percent each year. The sales S (in millions of dollars) for year t can be modeled by

$$S = 3723\left(\frac{6}{5}\right)^t$$

where $t = 0$ corresponds to 1994. Find the ratio of 1998 sales to 1995 sales.



53. **BASEBALL SALARIES** The average salary for a professional baseball player in the United States can be approximated by $y = 283(1.2)^t$, where $t = 0$ represents the year 1984. Using this approximation, find the ratio of an average salary in 1988 to an average salary in 1994.

FOCUS ON APPLICATIONS

ATLANTIC COD Adult Atlantic cod average about 3 feet in length and weigh from 10 to 25 pounds, though some may grow much larger.

ATLANTIC COD **In Exercises 54–56, use the following information. The average weight w (in pounds) of an Atlantic cod t years old can be modeled by the equation $w = 1.16(1.44)^t$.** ▶ Source: National Marine Fisheries Service, Northeast Science Center

54. Find the ratio of the weight of a 5-year-old cod to the weight of a 2-year-old cod. Express this ratio as a power of 1.44.

55. A 5-year-old cod weighs how many times as much as a 2-year-old cod?

56. According to the model, what is the average weight of an Atlantic cod when it is hatched? How did you get your answer?

57. **SALES** From 1995 through 1999, the sales for a national furniture store increased by about the same percent each year. The sales s (in millions of dollars) for year t can be modeled by $s = 476(1.13)^t$, where $t = 0$ represents 1995. Find the ratio of 1997 sales to 1999 sales.

FOCUS ON APPLICATIONS

ROWING Shells with 4 or 8 rowers usually have an additional nonrowing member of the team to direct the rest of the crew. This person is called the coxswain.

58. **OLYMPIC ROWING** The racing shells (boats) used in rowing competition usually have 1, 2, 4, or 8 rowers. Top speeds for racing shells in the Olympic 2000-meter races can be modeled by $s = 16.3(1.0285)^n$, where s is the speed in kilometers per hour and n is the number of rowers. Use the model to estimate the ratio of the speed of an 8-rower shell to the speed of a 2-rower shell.

59. **LEARNING SPANISH** You memorized a list of 200 Spanish vocabulary words. Unfortunately, each week you forget one fifth of the words you knew the previous week. The number of words S you remember after n weeks can be approximated by the following equation.

Vocabulary words remembered: $S = 200\left(\frac{4}{5}\right)^n$

a. Copy and complete the table showing the number of words you remember after n weeks.

Weeks, n	0	1	2	3	4	5	6
Words, S	?	?	?	?	?	?	?

b. CRITICAL THINKING How many weeks does it take to forget all but three words? Explain your answer.

60. **PROBABILITY CONNECTION** You roll a die eight times. What is the probability that you will roll eight sixes in a row?

61. **PROBABILITY CONNECTION** You roll a die six times. What is the probability that you will roll six even numbers in a row?

Test Preparation

MULTI-STEP PROBLEM In Exercises 62–65, use the following information. You work for a real estate company that wants to build a new apartment complex. A team is formed to decide in which state to build the complex. One team member wants to build in Arizona. Another team member wants to build in Michigan. Your boss asks you to decide where to build the complex.

62. You find that the population P of Arizona (in thousands) in 1995 projected through 2025 can be modeled by $P = 4264(1.0208)^t$, where $t = 0$ represents 1995. Find the ratio of the population in 2025 to the population in 2000.

INTERNET **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

63. You find that the population P of Michigan (in thousands) in 1995 projected through 2025 can be modeled by $P = 9540(1.0026)^t$, where $t = 0$ represents 1995. Find the ratio of the population in 2025 to the population in 2000.

▶ Source: U.S. Bureau of the Census

64. Which population is projected to grow more rapidly?

65. ***Writing*** Use the results from Exercises 62–64 to decide where to build the complex. Write a memo to your boss explaining your decision.

★ **Challenge**

66. **STACKING PAPER** A piece of notebook paper is about 0.0032 inch thick. If you begin with a stack consisting of a single sheet and double the stack 25 times, how tall will the stack be in inches? How tall will it be in feet? (*Hint:* Write and solve an exponential equation to find the height of the stack in inches. Then use unit analysis to find the height in feet.)

EXTRA CHALLENGE

www.mcdougallittell.com

MIXED REVIEW

POWERS OF TEN **Evaluate the expression.** **(Review 1.2, 8.2 for 8.4)**

67. 10^5 **68.** 10^3 **69.** 10^{-4} **70.** 10^{-8}

SKETCHING GRAPHS **Sketch the graph of the inequality in a coordinate plane.** **(Review 6.5)**

71. $x \geq 5$ **72.** $x + 3 < 4$ **73.** $y > -2$ **74.** $y \leq -1.5$

75. $x \geq 2.5$ **76.** $3x - y < 0$ **77.** $y \leq \frac{x}{2}$ **78.** $\frac{3}{4}x + \frac{1}{4}y \geq 1$

CHECKING FOR SOLUTIONS **Decide whether the ordered pair is a solution of the system.** **(Review 7.1)**

79. $2x + 4y = 2$
$-x + 5y = 13$ $(-3, 2)$

80. $x - 5y = 9$
$3x + y = 11$ $(1, -4)$

81. $8a + 4b = 6$
$4a + b = 3$ $\left(\frac{3}{4}, 0\right)$

82. $3c - 8d = 11$
$c + 6d = 8$ $\left(5, -\frac{1}{2}\right)$

SOLVING LINEAR SYSTEMS **Use linear combinations to solve the system.** **(Review 7.3)**

83. $x - y = 4$
$x + y = 12$

84. $-x + 2y = 12$
$x + 6y = 20$

85. $2a + 3b = 17$
$3a + 4b = 24$

QUIZ 1

Self-Test for Lessons 8.1–8.3

Evaluate the expression. **(Lessons 8.1, 8.2, and 8.3)**

1. $3^3 \cdot 3^4$ **2.** $(2^2)^4$ **3.** $[(8 + 2)^2]^2$ **4.** 7^{-4}

5. $4^{-3} \cdot 4^{-4}$ **6.** $\left(\frac{6}{7}\right)^{-1}$ **7.** $\frac{5^{-3}}{5^2}$ **8.** $\frac{3^4 \cdot 3^6}{3^3}$

9. $\left(\frac{5}{4}\right)^{-3}$ **10.** $\frac{(-2)^9}{(-2)^2}$ **11.** $6^0 \cdot \frac{1}{4^{-3}}$ **12.** $\frac{2^3 \cdot 2^{-4}}{2^{-3}}$

Simplify the expression. Write your answer with no negative exponents. **(Lessons 8.1, 8.2, and 8.3)**

13. $x^4 \cdot x^5$ **14.** $(-2x)^5$ **15.** $-\frac{3}{a^{-5}}$ **16.** $200^0 c^5$

17. $\frac{x^6}{x^4}$ **18.** $\frac{x^{-5}}{x^{-6}}$ **19.** $\left(\frac{-2m^2n}{3mn^2}\right)^4$ **20.** $x^4 \cdot \frac{1}{x^3}$

21. $(3a)^3 \cdot (-4a)^3$ **22.** $(8m^3)^2\left(\frac{1}{2}m^2\right)^2$ **23.** $\frac{20x^3y}{4xy^2} \cdot \frac{-6xy}{-x}$

24. **SAVINGS ACCOUNT** You started a savings account in 1994. The balance A is given by $A = 250(1.08)^t$, where $t = 0$ represents the year 2001. What is the balance in the account in 1994? in 1999? in 2001? **(Lesson 8.2)**

EXPLORING DATA AND STATISTICS

8.4

Scientific Notation

What you should learn

GOAL 1 Use scientific notation to represent numbers.

GOAL 2 Use scientific notation to describe **real-life** situations, such as the price per acre of the Alaska purchase in **Example 6**.

Why you should learn it

▼ To solve **real-life** problems, such as finding the amount of water discharged by the Amazon River each year in **Example 5**.

GOAL 1 USING SCIENTIFIC NOTATION

A number is written in **scientific notation** if it is of the form $c \times 10^n$, where $1 \leq c < 10$ and n is an integer.

ACTIVITY
Developing Concepts

Investigating Scientific Notation

1. Rewrite each number in decimal form.

   a. 6.43×10^4 **b.** 3.072×10^6 **c.** 4.2×10^{-2} **d.** 1.52×10^{-3}

2. Describe a general rule for writing the decimal form of a number given in scientific notation. How many places do you move the decimal point? Do you move the decimal point left or right?

EXAMPLE 1 *Rewriting in Decimal Form*

Rewrite in decimal form.

a. 2.834×10^2 **b.** 4.9×10^5 **c.** 7.8×10^{-1} **d.** 1.23×10^{-6}

SOLUTION

a. $2.834 \times 10^2 = 283.4$ **Move decimal point right 2 places.**

b. $4.9 \times 10^5 = 490{,}000$ **Move decimal point right 5 places.**

c. $7.8 \times 10^{-1} = 0.78$ **Move decimal point left 1 place.**

d. $1.23 \times 10^{-6} = 0.00000123$ **Move decimal point left 6 places.**

EXAMPLE 2 *Rewriting in Scientific Notation*

a. $34{,}690 = 3.469 \times 10^4$ **Move decimal point left 4 places.**

b. $1.78 = 1.78 \times 10^0$ **Move decimal point 0 places.**

c. $0.039 = 3.9 \times 10^{-2}$ **Move decimal point right 2 places.**

d. $0.000722 = 7.22 \times 10^{-4}$ **Move decimal point right 4 places.**

e. $5{,}600{,}000{,}000 = 5.6 \times 10^9$ **Move decimal point left 9 places.**

STUDENT HELP

HOMEWORK HELP
Visit our Web site www.mcdougallittell.com for extra examples.

To multiply, divide, or find powers of numbers in scientific notation, use the properties of exponents.

EXAMPLE 3 *Computing with Scientific Notation*

Evaluate the expression. Write the result in scientific notation.

a. $(1.4 \times 10^4)(7.6 \times 10^3)$

b. $(1.2 \times 10^{-1}) \div (4.8 \times 10^{-4})$

c. $(4.0 \times 10^{-2})^3$

SOLUTION

a. $(1.4 \times 10^4)(7.6 \times 10^3)$

$= (1.4 \cdot 7.6) \times (10^4 \cdot 10^3)$ **Associative property of multiplication**

$= 10.64 \times 10^7$ **Simplify.**

$= 1.064 \times 10^8$ **Write in scientific notation.**

b. $\dfrac{1.2 \times 10^{-1}}{4.8 \times 10^{-4}} = \dfrac{1.2}{4.8} \times \dfrac{10^{-1}}{10^{-4}}$ **Rewrite as a product.**

$= 0.25 \times 10^3$ **Simplify.**

$= 2.5 \times 10^2$ **Write in scientific notation.**

c. $(4.0 \times 10^{-2})^3 = 4^3 \times (10^{-2})^3$ **Power of a product**

$= 64 \times 10^{-6}$ **Power of a power**

$= 6.4 \times 10^{-5}$ **Write in scientific notation.**

....

Many calculators automatically switch to scientific notation to display large or small numbers. Try multiplying 98,900,000 by 500. If your calculator follows standard conventions, it will display the product using scientific notation.

4.945 E10 ← **Calculator display for 4.945 × 10^{10}**

STUDENT HELP

KEYSTROKE HELP
If your calculator does not have an EE function, you can enter the number in scientific notation as a product.
7.48 × 10 yˣ 7 +/-

EXAMPLE 4 *Using a Calculator*

Use a calculator to multiply 0.000000748 by 2,400,000,000.

SOLUTION

Enter 0.000000748 as 7.48×10^{-7} and 2,400,000,000 as 2.4×10^9.

KEYSTROKES	DISPLAY
7.48 EE 7 +/- × 2.4 EE 9 ENTER	1.7952 E3

▶ The product is 1.7952×10^3, or 1795.2.

FOCUS ON APPLICATIONS

AMAZON RIVER The Amazon River in Brazil contributes more water to Earth's oceans than any other river. Each second, it discharges 4.2×10^6 cubic feet of water into the Atlantic Ocean.

GOAL 2 SOLVING REAL-LIFE PROBLEMS

EXAMPLE 5 *Multiplying with Scientific Notation*

AMAZON RIVER How much water does the Amazon River discharge into the Atlantic Ocean each year?

SOLUTION

First find the number of seconds in a year. Then multiply the amount of water discharged per second by the number of seconds per year to find the total amount of water discharged into the Atlantic Ocean.

Use unit analysis to find the number of seconds in a year.

$$\frac{\text{days}}{\text{year}} \cdot \frac{\text{hours}}{\text{day}} \cdot \frac{\text{minutes}}{\text{hour}} \cdot \frac{\text{seconds}}{\text{minute}} = \frac{\text{seconds}}{\text{year}}$$

$$\frac{365}{1} \cdot \frac{24}{1} \cdot \frac{60}{1} \cdot \frac{60}{1} = 31{,}536{,}000 \approx 3.2 \times 10^7$$

There are about 3.2×10^7 seconds in a year.

Multiply to find the total amount of water discharged by the Amazon River into the Atlantic Ocean in one year.

$$\begin{aligned}\text{Total amount of water} &= \frac{4.2 \times 10^6 \text{ cubic feet}}{1 \text{ second}} \cdot \frac{3.2 \times 10^7 \text{ seconds}}{1 \text{ year}} \\ &= (4.2 \times 10^6) \cdot (3.2 \times 10^7) \\ &= 13.44 \times 10^{13} \\ &= 1.344 \times 10^{14}\end{aligned}$$

▶ One trillion is 1×10^{12}, so the total amount of water discharged into the Atlantic Ocean by the Amazon River is about 134 trillion cubic feet per year.

History

EXAMPLE 6 *Dividing with Scientific Notation*

In 1867, the United States purchased Alaska from Russia for \$7.2 million. The total area of Alaska is about 3.78×10^8 acres. What was the price per acre?

SOLUTION The price per acre is a unit rate.

$$\begin{aligned}\text{Price per acre} &= \frac{\text{Total price}}{\text{Number of acres}} \\ &= \frac{7.2 \times 10^6}{3.78 \times 10^8} \longleftarrow \textbf{7.2 million} = \mathbf{7.2 \times 10^6} \\ &= \frac{7.2}{3.78} \times 10^{-2} \\ &\approx 0.019\end{aligned}$$

▶ The price was about 2¢ per acre.

STUDENT HELP

Look Back
For help with unit rates, see p. 180.

GUIDED PRACTICE

Vocabulary Check ✓

1. Is the number 12.38×10^2 in scientific notation? Explain.

Concept Check ✓

2. Given the number 6.39×10^7, would you move the decimal point to the *left* or to the *right* to rewrite the number in decimal form?

Skill Check ✓

Rewrite in decimal form.

3. 4.3×10^2
4. 8.11×10^3
5. 2.45×10^{-1}
6. 9.38×10^5

Rewrite in scientific notation.

7. 39.6
8. 0.72
9. 1200
10. 0.0003
11. 6,900,000
12. 0.0000205
13. 72,000,000
14. 0.000000006

15. **ASTRONOMY** The distance between the ninth planet Pluto and the Sun is 5.9×10^9 kilometers. Light travels at a speed of about 3.0×10^5 kilometers per second. How long does it take light to travel from the Sun to Pluto?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

DECIMAL FORM **Rewrite in decimal form.**

16. 2.14×10^4
17. 98×10^{-2}
18. 7.75×10^0
19. 8.6521×10^3
20. 4.65×10^{-4}
21. 6.002×10^{-6}
22. 4.332×10^8
23. 1.00012×10^8
24. 1.1098×10^{10}

SCIENTIFIC NOTATION **Rewrite in scientific notation.**

25. 0.05
26. 95.2
27. 0.0422
28. 370.207
29. 700,000,000
30. 19.314
31. 0.008551
32. 2,730,000,000
33. 0.000459
34. 0.00032954
35. 88,000,000
36. 0.0000288

EVALUATING EXPRESSIONS **Evaluate the expression without using a calculator. Write the result in scientific notation and in decimal form.**

37. $(4 \times 10^{-2}) \cdot (3 \times 10^6)$
38. $(7 \times 10^{-3}) \cdot (8 \times 10^{-4})$
39. $(6 \times 10^5) \cdot (2.5 \times 10^{-1})$
40. $(1.2 \times 10^{-6}) \cdot (2.3 \times 10^4)$
41. $\dfrac{8 \times 10^{-3}}{5 \times 10^{-5}}$
42. $\dfrac{1.4 \times 10^{-1}}{3.5 \times 10^{-4}}$
43. $\dfrac{6.6 \times 10^{-1}}{1.1 \times 10^{-1}}$
44. $(3.0 \times 10^{-3})^2$
45. $(9 \times 10^3)^2$
46. $(3 \times 10^{-2})^4$

STUDENT HELP

HOMEWORK HELP
Example 1: Exs. 16–24
Example 2: Exs. 25–36
Example 3: Exs. 37–46
Example 4: Exs. 47–52
Example 5: Exs. 61, 62
Example 6: Exs. 58–60

CALCULATOR **Use a calculator to evaluate the expression. Write the result in scientific notation and in decimal form.**

47. $2{,}000{,}000 \cdot 12{,}000$
48. $6{,}000{,}000 \cdot 324{,}000$
49. $0.000279 \cdot 3{,}940{,}000{,}000$
50. $654{,}000 \cdot 0.000042$
51. $(2.4 \times 10^{-4})^2$
52. 0.000094^3

SCIENTIFIC NOTATION IN REAL LIFE In Exercises 53–57, write the number in scientific notation.

53. **LIGHTNING** The speed of a lightning bolt is 120,000,000 feet per second.

54. **WORLD POPULATION** In 1997, the population of the world was estimated at 5,852,000,000. **DATA UPDATE** of U.S. Bureau of the Census data at www.mcdougallittell.com

55. **ASTRONOMY** The star Sirius in the constellation Canis Major is about 50,819,000,000,000 miles from Earth.

56. **CHEMISTRY** The mass of a carbon atom is 0.000000000000000000000002 gram.

57. **SIZE OF JUPITER** Jupiter, the largest planet in our solar system, has a radius of about 4.4×10^4 miles. Use the equation $V = \frac{4}{3}\pi r^3$ to find Jupiter's volume.

STUDENT HELP

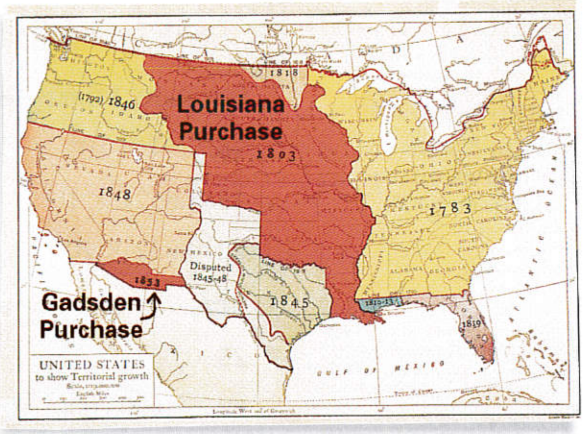
INTERNET APPLICATION LINK Visit our Web site www.mcdougallittell.com for more information about the purchase of Alaska, the Louisiana Purchase, and the Gadsden Purchase in Example 6 and Exs. 58–60.

HISTORY CONNECTION In Exercises 58–60, use the following information. In 1803, the Louisiana Purchase added 8.28×10^5 square miles to the United States. The cost of this land was \$15 million. In 1853, the Gadsden Purchase added 2.94×10^4 square miles, and the cost was \$10 million.

58. Find the average cost of a square mile for each of the purchases.

59. Find the average cost of an acre for each of the purchases. (*Hint:* There are 640 acres in a square mile.)

60. *Writing* Describe a factor that you think might explain the difference in the price per acre for these two purchases.



61. **WATERFALL** Stanley Falls in Congo, Africa, has an average flow of about 1.7×10^4 cubic meters per second. How much water goes over Stanley Falls in a typical 30-day month?

STUDENT HELP

INTERNET HOMEWORK HELP Visit our Web site www.mcdougallittell.com for help with Ex. 62.

62. **HEARTBEATS** Consider a person whose heart beats 70 times per minute and who lives to be 85 years old. Estimate the number of times the person's heart beats during his or her life. Do not acknowledge leap years. Write your answer in decimal form and in scientific notation.

63. **TELEPHONE SURVEY** Use the table which shows the population and the number of local telephone calls made in five states in 1994 to find the number of local calls made per person in each state.

State	Local Calls	Population
Texas	3.9×10^{10}	1.8×10^7
Minnesota	7.0×10^9	4.6×10^6
Pennsylvania	1.9×10^{10}	1.2×10^7
Vermont	4.7×10^8	5.8×10^5
California	5.6×10^{10}	3.1×10^7

▶ Source: U.S. Bureau of the Census

Test Preparation

64. MULTIPLE CHOICE Which number is not in scientific notation?

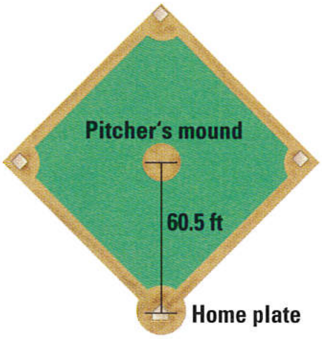
Ⓐ 1×10^4 Ⓑ 3.4×10^{-3} Ⓒ 9.02×10^2 Ⓓ 12.25×10^{-5}

65. MULTIPLE CHOICE Evaluate $\frac{1.1 \times 10^{-1}}{5.5 \times 10^{-5}}$ using scientific notation.

Ⓐ 0.2×10^{-4} Ⓑ 2.0×10^4 Ⓒ 2.0×10^3 Ⓓ 0.2×10^4

★ Challenge

66. BASEBALL A baseball pitcher can throw a ball to home plate in about 0.5 second. The distance between the pitcher's mound and home plate is 60.5 feet.



a. Fill in the missing numbers and simplify the following expression to find the rate, in scientific notation, at which the ball is traveling in millimeters per second.

$$\frac{?\text{ feet}}{?\text{ second}} \cdot \frac{1\text{ meter}}{3.3\text{ feet}} \cdot \frac{?\text{ millimeters}}{1\text{ meter}} \approx \underline{?} \times 10^{?}\text{ millimeters per second}$$

b. To hit a home run, a batter has a leeway of about 200 millimeters in the point of contact between the bat and the ball. What is the most the batter's timing could be off and still hit a home run? Explain your calculations. (*Hint:* Find the time it takes the ball to travel 200 millimeters.)

c. CRITICAL THINKING You are at bat. The pitcher throws you a pitch. Your timing is off by 0.006 second. Could you hit a home run on this pitch? Explain your answer.

EXTRA CHALLENGE
www.mcdougallittell.com

MIXED REVIEW

PERCENTS AS DECIMALS Write the percent as a decimal. (Skills Review page 784 for 8.5)

67. 22% **68.** 87.5% **69.** 0.07% **70.** 8.42%

71. $\frac{1}{2}\%$ **72.** $\frac{3}{4}\%$ **73.** 255% **74.** $1\frac{1}{4}\%$

GRAPHING LINEAR SYSTEMS Use the graphing method to solve the linear system and describe its solution(s). (Review 7.5)

75. $4x + 2y = 12$
$-6x + 3y = 6$

76. $3x - 2y = 0$
$3x - 2y = -4$

GRAPHING Graph the system of linear inequalities. (Review 7.6)

77. $2x + y \leq 1$
$-2x + y \leq 1$

78. $x + 2y < 3$
$x - 3y > 1$

79. $2x + y \geq 2$
$x \leq 2$

SIMPLIFYING EXPRESSIONS Simplify the expression. (Review 8.2 and 8.3)

80. 2^{-4} **81.** $\left(\frac{1}{10}\right)^{-3}$ **82.** $\frac{1}{(2x)^{-2}}$ **83.** $\frac{7^4 \cdot 7}{7^7}$

ACTIVITY 8.5

Developing Concepts

Group Activity for use with Lesson 8.5

Linear and Exponential Growth Models

SET UP
Work in a small group.

MATERIALS
graph paper

QUESTION

How are linear growth models and exponential growth models different?

EXPLORING THE CONCEPT

1. The equation $y = 5x + 20$ is a *linear growth model*. Copy and complete the table.

x	0	1	2	3	4	5
y	20	25	?	?	?	?

2. Graph $y = 5x + 20$.

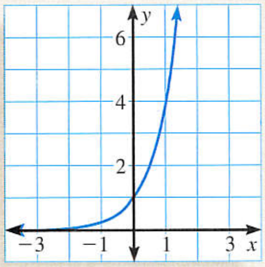
3. The equation $y = 5^x$ is an *exponential growth model*. Copy and complete the table.

x	0	1	2	3	4	5
y	1	5	?	?	?	?

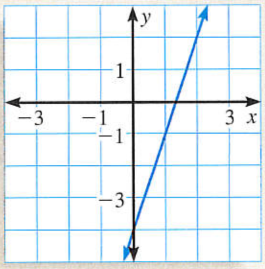
4. Graph $y = 5^x$.

5. Which of the graphs below shows a *linear growth model*? Which shows an *exponential growth model*? Explain how you know.

a.



b.



DRAWING CONCLUSIONS

In Exercises 1–6, identify the equation as a *linear* growth model or an *exponential* growth model.

1. $y = x + 5$

2. $y = 3^x$

3. $y = 10 + 2x$

4. $y = 15 + 2^x$

5. $y = 5(4x - 7)$

6. $y = 10(1.2)^x$

7. Look at your data and graph in Steps 1 and 2 to complete the statement.

A linear growth model increases the _?_ *amount for each unit on the x-axis.*

8. Describe the rate of increase in an exponential growth model.

9. **CRITICAL THINKING** You accept a job that pays \$20,000 your first year. Would you rather receive a raise of \$500 each year or a raise of 3% of your current salary each year? Does your answer depend on how long you plan to stay at the job? Explain your reasoning.